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Mathematical model of fractal conduits flow mechanics in the Gunungsewu karst area, Yogyakarta Special Region, Indonesia

Sari Bahagiarti Kusumayudha*

Geology Department,
Faculty of Mineral Technology,
Universitas Pembangunan Nasional Veteran Yogyakarta,
Jl Padjadjaran, Condongcatur, Yogyakarta, Indonesia
Email: saribk@upnyk.ac.id
*Corresponding author

M.T. Zen and Sudarto Notosiswoyo

Institut Teknologi Bandung,
Jl Ganesha, 10, Bandung, Indonesia
Email: zenorca@ibm.net
Email: sudarto@mining.itb.ac.id

Rudy Sayoga Gautama

Mining Engineering Department,
Faculty of Mining Engineering and Petroleum Engineering,
Institute of Technology of Bandung,
Jl Ganesha, 10, Bandung, Indonesia
Email: r_sayoga@mining.itb.ac.id

Abstract: The Gunungsewu area, Yogyakarta, Indonesia is known as a karst landform with unique hydrogeological setting. Water is stored and flown in subsurface chambers and channels. There are some underground rivers, with flow rate ranging from 200 l/sec to 1,000 l/sec, used by the local people to fulfil their fresh water needs. The Gunungsewu is often exploited as a research area by karst geologists, geomorphologists, and hydrogeologists due to its fascinating natural features. A variety of groundwater modelling has been developed, including mathematical modelling, but in this case, the water is generally illustrated as flowing through a pipe with simplified channel shape. In fact, the form of karst channels is actually uneven. This irregularity causes karst channels should be described as fractal geometry. Consequently, some groundwater flow mechanics equations need to be modified, so that they are more appropriate to be applied in the karst region.

Keywords: mathematical model; groundwater flow; fractal geometry; karst.

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Biographical notes: Sari Bahagiarti Kusumayudha is a Professor of Engineering Geology and Hydrogeology at the Geological Engineering Department, Faculty of Mineral Technology, UPN 'Veteran' Yogyakarta. She completed many publications related to the field of hydrogeology, karst hydrogeology, engineering geology and geological disasters in various conferences, scientific journals both in Indonesia and international. Some popular scientific books and reference books on fractal analysis for hydrogeological exploration, fractal analysis for earthquake and tsunami mitigation, hydrogeology, karst hydrogeology, surface water, active volcanoes, and Merapi Volcano of Indonesia have also been written by her.

M.T. Zen is an Emeritus Professor from the Geophysical Engineering Department of the Bandung Institute of Technology. He is known as a geologist, geophysicist, public intellectual and natural explorer. His life is inseparable from the world of literature. He has written and published many books, especially in the environmental field. Among others of his books are entitled, *Towards Environmental Conservation*, and *The Boundaries of Growth*. When he was still active as a professor, he has produced many qualified and credible young doctors and professors.

Sudarto Notosiswoyo is an Emeritus Professor from the Mining Engineering Study Program, Bandung Institute of Technology (ITB), in the Scientific Group of Earth Resources Exploration. He has ever served as the dean of the Faculty of Mineral Technology, ITB. His publications in the form of articles in journals, both national and international, generally cover the fields of mining exploration and hydrogeology.

Rudy Sayoga Gautama is a Lecturer and researcher and Professor in the Mining Engineering Department, Bandung Institute of Technology (ITB). He included in Mining Environment Expertise Division. He has ever served as the Chair of the Institute for Research and Community Service. One of his publications is entitled *Development of a New Covering Strategy in Indonesian Coal Mines to Control Acid Mine Drainage Generation: A Laboratory-Scale Result.*

1 Introduction

In the Yogyakarta Special Region, Central Java, Indonesia, there is a very well-known karst landform, included in one of UNESCOs Global Geoparks, namely Gunungsewu area (Figure 1). Scientists, experts, and students either from Indonesia or other countries often make this area as the locality of research, especially in the fields of geomorphology and hydrogeology (Haryono et al., 2016). As known, hydrogeology of the Gunungsewu area is characterised by secondary porosities and subsurface drainage systems through underground rivers that are used by people to fulfil their freshwater needs (Kusumayudha, 2004).

Groundwater flow type of karst system is generally classified into conduit flow through cavity networks that are commonly irregular in shape (Kusumayudha, 2005). This means that Darcy's law, although has been worked out in detail by White (1988) cannot be appropriately applied to explore the groundwater (Kusumayudha et al., 1997; Kusumayudha, 2004). Related to hydrogeological channel flows, rules considered to be utilised are Bernoulli and Hagen-Poiseuille equations (White, 1988). All this time, the Bernoulli and Hagen-Poiseuille equations determine that the cross-sectional shape of a conduit is circular (White, 1988). In fact, the form of karst channels is not exactly like a pipe, but tends to be uneven (Kusumayudha et al., 1997; Kusumayudha, 2005). On the other hand, an object which shows a complicated structure can be generally classified into fractal geometry, that have non-integer value of dimension, called fractal dimension (Mandelbrot, 1983).

The use of fractal geometry in regular porous media has often been discussed by hydrogeologists (Adler, 1996), but its application specifically in the field of karst hydrogeology has not been widely studied (Kusumayudha, 2004, 2009). On the other side, the Bernoulli equation which regulates the mechanics of water flow through a conduit generally simplifies its cross-section as a Euclidean geometry. In fact, conduits developing karst aquifers are to be considered more appropriately as fractal objects (Kusumayudha, 2005). Actually, fractal geometry concept promises a power to unravel tangled threads that are associated to complexity (Adler, 1996; Sahimi and Yortsos, 1990). Based on the backgrounds mentioned above, this study was done with the objective to develop a mathematical model of the flow mechanics in Gunungsewu karst area, based on the fractal geometry approach. As it is widely applied, hydrogeological modelling is one of practical approaches for recharge and discharge estimation of sustainable groundwater management (Woldie and Herath, 2015).

Figure 1 Location of the study area (see online version for colours)

2 Approaches and materials of the study

This study methodologically unites field surveys, mapping, and literature reviews. Data to be analysed were derived from field survey and mapping, combined with secondary data from previous studies. The approach is carried out by referring to the theoretical basis of karst hydrology and fractal geometry. Completing the method, fractal geometry analysis was applied to develop the mathematical model of flow mechanics in the study area.

2.1 Karst

Karst is a landscape that is characterised by the presence of closed depressions with various sizes and arrangements, disturbed surface flow patterns, caves, and underground drainage systems (White, 1988). Hydrologically, karst is a system where meteoric water interacts with soluble rock formations in open schemes (Esteban, 1996). The karst environment is generally unique and complex with typical hydrogeology and geomorphology.

The formation of the karst system requires an initial permeability network, consisting of porosity, and cracks, which are not formed during the karstification process, but increased by corrosion and mechanical erosion, or eliminated by cementation and sedimentation (Moore, 1989). The most influential factors in the development of karst are pH and temperature of the meteoric water available for corrosion and erosion, hydraulic head, and hydraulic gradient between the catchment area and the discharge area (Esteban, 1996).

2.2 Fractal geometry

Fractal, from the origin of the word *fractus*, means broken. It is an object that has a fractal dimension greater than the dimensions of its topology (Mandelbrot, 1983; Bunde and Havlin, 1994). This object is formed by the parts of the wake, in the same way, iteratively, or by interlocking each other (Peitgen et al., 1992). The characteristics of fractal objects cannot be categorised as a euclidean geometry which is described by its integer value of dimensions such as one (1), two (2) and three (3). In contrast, the dimensions of fractal objects are expressed as non-integer numbers, for example 0.97, 1.34, 2.85, etc. (Mandelbrot, 1983).

In fractals, a part of an object is a simple form of the whole (the part is reminiscent of the whole). Fractal geometry shows properties of self-similarity, self-affinity, and scale-invariant (statistically as well as in general). Self-similar properties indicate that a fractal object is arranged by parts that are similar to each other (Figure 2). Self-affine means fractal objects are arranged by parts that are intertwined with each other. While scale invariant means that fractals do not have a specific length scale (Mandelbrot, 1983). Related to the properties mentioned above, the fractal concept is able to decipher the construction of a complex natural object, into primitive elements (Peitgen et al., 1992).

Natural fractal objects rarely have self-similar properties. In general, they only have a statistically similar self-nature, therefore it is called statistical self-similar fractal or statistical self-affine fractal. An example of such a fractal object is fractional Brownian motion (fBm). This is a trace of the movement of a particle in a liquid, which is a fractal behaviour curve. Another statistical fractal example is fractional Gaussian noise (fGn). If the fBm curve is area, then the fGn curve is spatial (Sahimi and Yortsos, 1990).

Determination of fractal dimensions is very important in solving practical quantification problems, because they are often related to the natural processes that take place (Korvin, 1992). Because fractals are generally irregular in shape, the degree of irregularity can be quantified using their dimension values (Sukmono, 1996). Many ways can be done to determine fractal dimensions, including sand box, box counting, balls covering, cantor dust, and similarity methods (Peitgen et al., 1992).

Figure 2 Koch curve, a similar fractal, has a dimension = 12,618

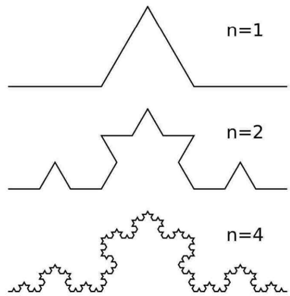
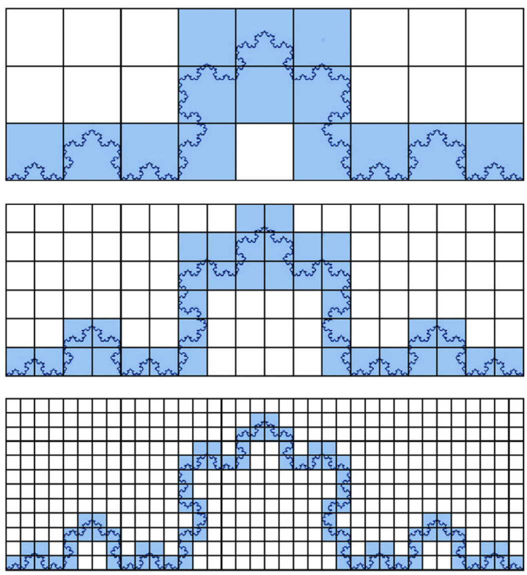


Figure 3 Example of box counting method to obtain fractal dimension (see online version for colours)



In this study the technique used to determine the fractal dimension is the box counting method. The box counting method can be applied to multiple objects classified as statistical self-similar or statistical self-affine fractal. The box counting method is done by creating a certain side grid (*r*) in fractal objects (Figure 3). Furthermore, the fractal dimension (*D*) is determined by the equation (Tricot, 1995):

$$D = \lim_{r \to 0} \frac{\log Nr(F)}{-\log r} \tag{1}$$

$Nr(F)$ the number of boxes that intersects the fractal set (F)

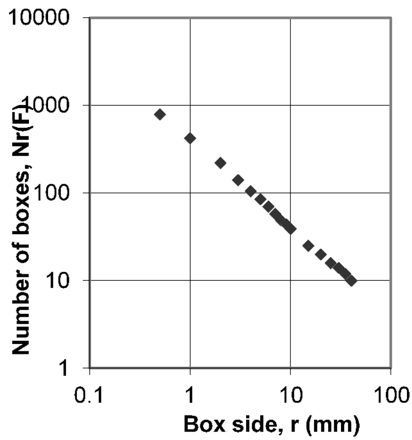
r the size of box side.

Calculation of $Nr(F)$ is done repeatedly by changing the size of r to the smallest possible (close to 0). Then the variations of $Nr(F)$ number to r can be plotted into the log-log graph. The fractal dimension is manifested in the slope of the plot. (Figure 3, Figure 4).

Calculated using the box counting method, with the side length of box = r, and $Nr(F)$ is the number of squares surrounding F, then the length of the fractal curve becomes:

$$L(F) = \lim_{r \to 0} Nr(F) \tag{2}$$

Figure 4 Example of plot of Nr(F) versus r on a log-log graph for determining fractal dimension, D



Source: Kusumayudha et al. (2000)

3 Results and discussion

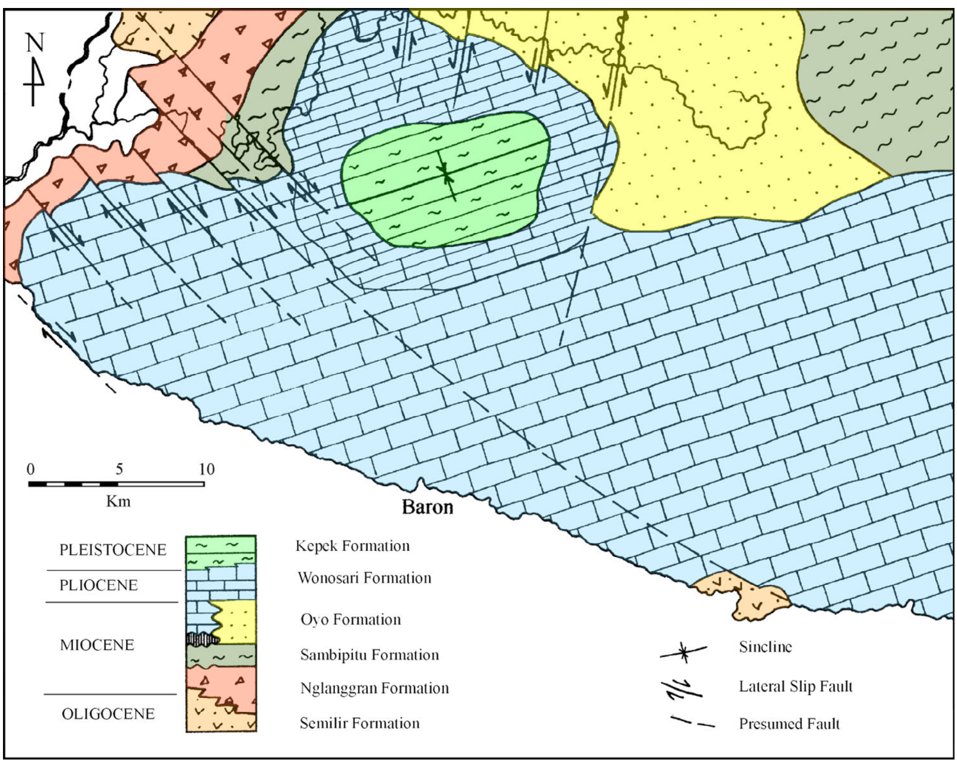
3.1 Karst hydrogeology of the Gunungsewu area

The Gunungsewu area is representing cone karst hills (Figure 5), stratigraphically composed of a volcanic rock group, Oligocene to Miocene aged, and a carbonate rocks group deposited during Miocene. The volcanic rocks group comprises tuffaceous sandstone of Semilir Formation, volcanic breccia of Nglanggeran Formation, and tuffaceous marl of Sambipitu Formation. While the carbonates group consists of calcarenite and calcareous sandstone of Oyo Formation, reef and bedded limestone of Wonosari Formation, and marl of Kepek Formation (Figure 6). Lithology which performs karst topography is dominated by Wonosari Formation.

In the Gunungsewu area, there are two types of groundwater flow, namely diffuse flow and conduit flow. Diffuse flow occurs in intergrained media, represented by chalky limestone, with openings (diameter size is in millimetres or smaller) (Kusumayudha, 2004, 2009). Whereas conduit flow exists in irregular shaped ditches of diameter sizes from centimetres to meters, expressed as conduits and caves of karst (Figure 7, Figure 8). Groundwater that supplies the fresh water need of the people in the surrounding areas mainly gotten from a subsurface river that is governed by conduit flow hydrogeological regulations.

Figure 5 Cone karst hills of the Gunungewu area (see online version for colours)

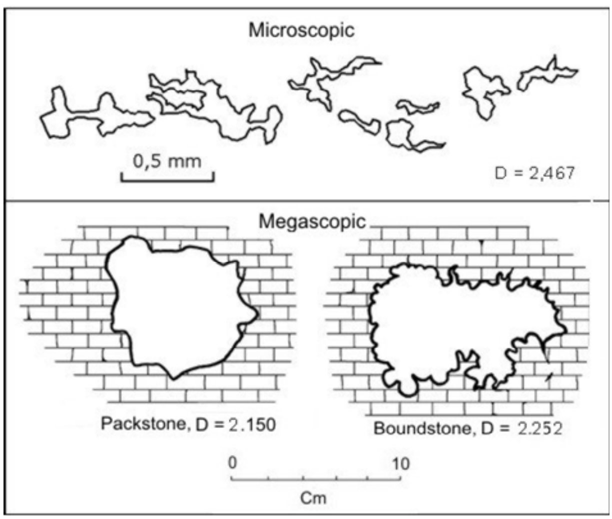
Figure 6 Geologic map of the Gunungewu area (see online version for colours)



Source: Kusumayudha et al. (2000) and Kusumayudha (2005)

Figure 7 Karstic limestone with conduits for meteoric water to flow (see online version for colours)

Figure 8 Cross sectional conduits of the Gunungsewu area (see online version for colours)



Source: Kusumayudha (2005)

3.2 *Flow mechanics in fractal conduits*

The permeability networking in karst aquifers often show fractal properties. Groundwater flowing in a karst conduit uses the Earth's gravitational force as an energy source. In the steady and frictionless flow of a closed system, the total energy will be equal to the amount of kinetic energy and potential energy of the system. This is governed by the Bernoulli equation (Fetter, 1994), as follows:

$$gz + \frac{P}{\rho} + \frac{v^2}{2} = constant \tag{3}$$

g Gravity acceleration

z The potential energy generated by the elevation, z

P The pressure acting on the system

v Velocity

r Fluid density.

In this case, the potential energy is expressed as hydraulic head, h. The kinetic energy of this system is comprehended in the water movement as reflected in its velocity, v. If it is divided by the gravity acceleration, g, the Bernoulli equation, becomes:

$$h + \frac{P}{\rho .g} + \frac{v^2}{2g} = h\tau \tag{4}$$

$h\tau$ total head

P pressure (head).

where the numbers of elevation head, pressure head, and velocity head are equal to the total head, $h\tau$, in the system.

Flow in a karst conduit is not frictionless (White, 1988). The relationship between the friction head value, hf, which is the energy lost from the friction between the water and the conduit wall, is reflected in the energy balance equation between inlet and outlet of the conduits, as another form of the Bernoulli equation:

$$h\tau = \frac{P_1}{\rho .g} + \frac{v_1^2}{2g} + h_1 = \frac{P_2}{\rho .g} + \frac{v_2^2}{2g} + h_2 + hf \tag{5}$$

P_1 Pressure head of inlet

P_2 Pressure head of outlet

v_1 Velocity of inlet

v_2 Velocity of outlet.

Groundwater flows in a closed karst conduit; there work two kinds of forces, namely the inertial forces that are associated with the momentum of moving water masses, and the viscous forces that occur because the fluid layers slide one to another (White, 1988). The important relationship between inertial and viscous forces is expressed as the ratio between the two things, a number without units, called the Reynolds number, NR (Fetter, 1994), and the equation becomes:

$$\frac{F_t}{F_v} = \frac{\rho .v^2 / R}{\eta .v / R^2} = \frac{\rho .vR}{\eta} = N_R \tag{6}$$

F_i Fluid inertial force

F_v Fluid viscous force

η Fluid viscosity

R Hydraulic radius

NR Reynolds number.

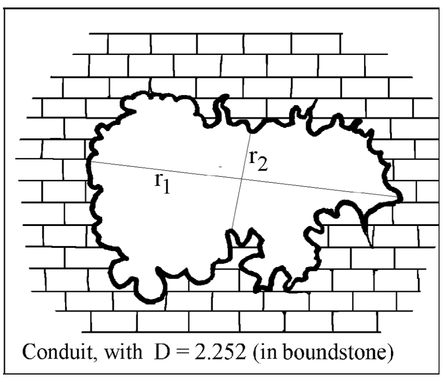
Two flow regimes are said to be similar, if they have equal Reynolds number, regardless of the other parameters (White, 1988). For a circular cross section conduit, R is the diameter, $R = 2r$. The dimension of circumference of a circle is 2 (two), while the dimension of its radius is 1 (one), or 1/2 of the dimension of the circumference. This states that the magnitude of the radius of a circle or a circular fractal object is influenced by its circumference. In a circle, $R = 2r^{1/2\times 2}$ or $R = 2r^{1}$. So, in a fractal conduit with irregular cross-sectional shape, to get the value of R can be approached by:

$$R = (r_1 + r_2)^{1/2D} \tag{7}$$

D is the fractal dimension of the perimeter of the conduit cross section, while r_1 and r_2 are the longest and the shortest radius respectively (Figure 9). So that equation (7) develops into a mathematical model of a fractal conduit flow as:

$$\frac{F_t}{F_v} = \frac{\rho.v^2 / (r_1 + r_2)^{1/2D}}{\eta.v / (r_1 + r_2)^{D}} = \frac{\rho.v (r_1 + r_2)^{1/2D}}{\eta} = N_R \tag{8}$$

Figure 9 Conduit cross section with longest (r1) and shortest (r2) radius



A fluid flow performance depends on its density ρ, and viscosity (thickness), η, both of which are the function of temperature (Table 1).

Table 1 Physical properties of water

Temperature (ºC)	*Viscosity, η (poise)*	*Density, ρ (g.ml⁻¹)*
0.0	0.01787	0.99987
5.0	0.01519	0.99999
10.0	0.01307	0.99973
15.0	0.01139	0.99913
20.0	0.01002	0.99823
25.0	0.008904	0.99707
30.0	0.007975	0.99567
35.0	0.007194	0.99406
40.0	0.006529	0.99224

Source: White (1988)

In such a water that moves slowly through the smooth surface of a pipe, the flow regime is assumed to be laminar. If the speed of motion increases, the flow lines become unstable. The irregularity of the pipe wall causes more widespread interference, then the flow regime will change from laminar to be turbulent (White, 1988).

Hagen-Poiseuille equation defines total discharge of flow in a conduit:

$$Q = \frac{\pi \rho r^2}{8\eta}\frac{dh}{dl} \tag{9}$$

Referring to equations (7) and (8), the total discharge through a fractal channel becomes to be:

$$Q = \frac{\pi.\rho.g\left(\frac{r_1 + r_2}{2}\right)^D}{8\eta}\frac{dh}{dl} \tag{10}$$

Q Total discharge

D Fractal dimension

dh/*dl* Hydraulic gradient.

In order to apply the mathematical model that has been built in this study, four subsurface rivers have been selected, namely Kali Bribin, Kali Seropan, Kali Suci and Kali Ngobaran as the samples. Based on conventional measurements using current metre, the rivers have flow rates ranging 200 to 1,000 l/second (Kusumayudha, 2009, 2013), and are used by the surrounding community to meet their clean and fresh water needs. By using the flow mechanics mathematical model discussed above, results of total discharge computation on samples of the study area are as follows (Table 2).

Table 2 Results of total discharge computation for samples in the study area, using $\rho = 0.99$, $\eta = 0.01$, g = 9.807 m/s2

Subsurface channel name	*Hydraulic gradient (dh/dl)*	*Fractal dimension (D)*	*Radius of the channel (r1, r2) (m)*	*Total discharge, Q conventional (l/sec)*	*Q using new model (l/sec)*
Kali Bribin	0.105	1.007 ± 0.01	0.80, 11.0	800–1000	2,405
Kali Seropan	0.12	1.003 ± 0.01	0.75, 5.50	600–800	1,322
Kali Suci	0.09	1.028 ± 0.01	0.70, 2.80	400–500	613
Kali Ngobaran	0.07	1.011 ± 0.01	0.60, 2.50	200–300	418

Data used for the computation were taken in the mouth or inside of the cave, where the cross-sectional shape of the underground rivers really represent the fractal geometry (Figure 10). In this condition, the mathematical model of flow mechanics in fractal conduit can be applied instead of the conventional flow model.

The results of the total discharge calculation show a greater number than the results of conventional measurements. This is due to quantification of the channel hydraulic radius which uses the assumption for maximum discharge in the peak of the rainy season. Meanwhile the conventional measurements were carried out in the dry season. However, the results of this study are objected to be an alternative method which can be applied when the target of research is not a usual entity but a fractal object. Therefore, it is only natural that the methods applied need to get a development tailored to the conditions of the field. Thus, the results are expected to be more accurate, and surely still opened for further development.

Figure 10 Kali Suci stream entering the mouth of an underground river, showing uneven shaped of channel, can be categorised as fractal geometry (see online version for colours)

4 Conclusions

From the discussion in this article, the following can be concluded:

- Gunungsewu is a karst area with unique, irregular, and disorder hydrogeological characteristics. Related to this condition, groundwater flow mechanics should be governed by a developed mathematical model different from the previous conventional model.
- The existence of fractal geometries in karst landform should not be ignored, as well as its presence in the permeability network of the Gunungsewu hydrogeological system. Therefore, to get more accurate results, the mathematical equations of groundwater flow mechanics in the karst area need to be improved by accommodating fractal dimension into the formulas.
- The assessment elaborated in this study represents a development of the existing approaches, and remains open for further improvement. The results are expected to open new ideas and perspectives in hydrogeological studies or research for the advancement of this field of science in the future.

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