

Proceedings of the 3rd International Conference on Mathematical Sciences



Kuala Lumpur, Malaysia
17–19 December 2013

Editors

Wan Zawiah Wan Zin, Syahida Che Dzul-Kifli, Fatimah Abdul Razak and Anuar Ishak

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Table of Contents

Preface: 3rd International Conference on Mathematical Sciences Wan Zawiah Wan Zin, Syahida Che Dzul-Kifli, Fatimah Abdul Razak, and Anuar Ishak	1
APPLIED MATHEMATICS	
A permutation parallel algorithm under exchange restriction with message passing interface Sharmila Karim, Zurni Omar, and Haslinda Ibrahim	3
Bifurcation of rupture path by linear and cubic damping force Dennis L. C. C., Chew X. Y., and Lee Y. C.	10
Block multistep methods based on rational approximants Teh Yuan Ying, Zurni Omar, and Kamarun Hizam Mansor	15
Boundary layer flow and heat transfer past a shrinking sheet in a copper-water nanofluid Nur Liyana Aleng, Norfifah Bachok, Norihan Md. Arifin, and Anuar Ishak	22
Challenging of path planning algorithms for autonomous robot in known environment R. N. Farah, N. Irwan, Raja Lailatul Zuraida, Umairah Shaharum, and Hafiz Mohd Hanafi@Omar	29
Characterization of essential proteins based on network topology in proteins interaction networks Sakhinah Abu Bakar, Javid Taheri, and Albert Y. Zomaya	36
Comparing linear and nonlinear differential equations of differential transformation method by other numerical methods Che Haziqah Che Hussin, Adem Kilicman, and Arif Mandangan	43
Compression-RSA technique: A more efficient encryption-decryption procedure Arif Mandangan, Loh Chai Mei, Chang Ee Hung, and Che Haziqah Che Hussin	50
Estimation of information theoretic measures on the Ising model Fatimah Abdul Razak and Henrik Jeldtoft Jensen	56

Flood control project selection using an interval type-2 entropy weight with interval type-2 fuzzy TOPSIS Nurnadiyah Zamri and Lazim Abdullah	62
Fourth-order solutions of nonlinear two-point boundary value problems by Newton-HSSOR iteration Jumat Sulaiman, Mohd. Khatim Hasan, Mohamed Othman, and Samsul Ariffin Abdul Karim	69
Free convection boundary layer flow near the lower stagnation point of a solid sphere with convective boundary conditions in a micropolar fluid Hamzeh Taha Alkawasbeh, Mohd Zuki Salleh, Razman Mat Tahar, Roslinda Nazar, and Ioan Pop	76
Functional data analysis on ground reaction force of military load carriage increment Wan Rozita Wan Din and Azmin Sham Rambely	83
Hopf bifurcation in Hutchinson's equation with distributed delay I. Darti	89
Improved nonlinear prediction method Nur Hamiza Adenan and Mohd Salmi Md Noorani	94
Kinematic analysis of rope skipper's stability Nor Atikah Ab Ghani and Azmin Sham Rambely	100
Linear fixed point function for solving system of polynomial equations Hafizudin Mohamad Nor, Ahmad Izani Md. Ismail, and Ahmad Abdul Majid	105
Mathematical modelling for the drying method and smoothing drying rate using cubic spline for seaweed <i>Kappaphycus Striatum</i> variety <i>Durian</i> in a solar dryer M. K. M Ali, M. H. Ruslan, M. S. Muthuvalu, J. Wong, J. Sulaiman, and S. Md. Yasir	113
Maximum number of limit cycles of a general Lienard equation Hero WaisiSalih and Zainal Abdul Aziz	121
MHD stagnation-point flow and heat transfer over a permeable stretching/shrinking sheet Fadzilah Md Ali, Roslinda Nazar, Norihan Arifin, and Ioan Pop	125

Mixed convection boundary-layer flow near the stagnation-point on a vertical surface in a nanofluid	132
Anisah Dasman, Noor Adila Othman, Salimah Ahmad, Nor Azizah Yacob, and Anuar Ishak	
Mixed convection flow about a solid sphere with constant heat flux embedded in a porous medium filled by a nanofluid: Buongiorno-Darcy model	139
Leony Tham, Roslinda Nazar, and Ioan Pop	
Modified multistep method based on interpolation for solving ordinary differential problem	146
Azman Ismail, Rokiah@Rozita Ahmad, Ummul Khair Salma Din, and Mohd Rosli A. Hamid	
Natural convection and viscous dissipation in a square porous enclosure using a thermal nonequilibrium model	151
Abeer Alhashash, Habibis Saleh, and Ishak Hashim	
New results on the W-weighted Drazin inverse	157
M. Nikuie and M. Z. Ahmad	
Numerical investigation of mixed convection on a stagnation point flow past a stretching vertical surface with convective boundary conditions	163
M. K. A. Mohamed and M. Z. Salleh	
Numerical solution of differential algebraic equations (DAEs) by mix-multistep method	170
Yong Faezah Rahim, Mohamed Suleiman, and Zarina Bibi Ibrahim	
Numerical solution of the free convection boundary layer flow over a horizontal circular cylinder with convective boundary conditions	179
Norhafizah Md Sarif, Mohd Zuki Salleh, Razman Mat Tahar, and Roslinda Nazar	
Numerical solutions for linear Fredholm integral equations of the second kind using 2-point half-sweep explicit group method	186
Mohana Sundaram Muthuvalu, Sarat Chandra Dass, Beh Hoe Guan, Dennis Ling Chuan Ching, and Jumat Sulaiman	
Numerical solutions of three-dimensional boundary layer flow and heat transfer past a permeable shrinking surface in a Cu-water nanofluid	192
Mohd Amin Mohd Noor, Roslinda Nazar, Khamisah Jafar, and Ioan Pop	

On oscillation of nonlinear second order differential equations Ambarka Abdalla Salhin, Ummul Khair Salma Din, Rokiah Rozita Ahmad, and Mohd Salmi Md Noorani	199
Perona Malik anisotropic diffusion model using Peaceman Rachford scheme on digital radiographic image Suhaila Abd Halim, Rohayu Abd Razak, Arsmah Ibrahim, and Yupiter HP Manurung	208
Prediction of single stage limit language and adult language via Yusof-Goode approach Wen Li Lim, Yuhani Yusof, and Norhayati Rosli	215
Prediction on carbon dioxide emissions based on fuzzy rules Herrini Pauzi and Lazim Abdullah	222
Radiation and porosity effects on the magnetohydrodynamic flow near a vertical plate that applies shear stress to the fluid with mass diffusion Arshad Khan, Ilyas Khan, and Sharidan Shafie	227
Roles of the influential parameters in the incineration process using centrality concept of graph theory W. A. Awatif, B. Sabariah, M. Rashid, and M. Normah	233
Runge-Kutta method for wall shear stress of blood flow in stenosed artery Izyan Syazana Awaludin and Rokiah@Rozita Ahmad	240
Separability criteria and method of measurement for entanglement Siti Munirah Mohd, Bahari Idrus, and Muriati Mukhtar	246
Some relations between two stages DNA splicing languages Mohammad Hassan Mudaber, Yuhani Yusof, and Mohd Sham Mohamad	254
Steady fluid flow and heat transfer over an exponentially shrinking sheet with partial slip Rajesh Sharma, Anuar Ishak, and Ioan Pop	260
Stagnation-point flow over a nonlinearly stretching/shrinking sheet in a micropolar fluid Nor Azizah Yacob and Anuar Ishak	266

A new steepest descent method Zubai'ah Zainal Abidin, Mustafa Mamat, Mohd Rivaie, and Ismail Mohd	273
The double scale convergence for problem of dissipative evolution in periodically perforated environment Dumitrache Mihaela and Gheldiu Camelia	279
The global convergence properties of a conjugate gradient method Osman Omer, Mustafa Mamat, Abdelrhman Abashar, and Mohd Rivaie	286
The proof of sufficient descent condition for a new type of conjugate gradient methods Abdelrhman Abashar, Mustafa Mamat, Mohd Rivaie, Ismail Mohd, and Osman Omer	296
Unscented Kalman filtering for wave energy converters system identification Mohd Aftar Abu Bakar, David A. Green, Andrew V. Metcalfe, and Noratiqah Mohd Ariff	304
Unsteady boundary layer flow and heat transfer over a stretching sheet with a convective boundary condition in a nanofluid Syahira Mansur and Anuar Ishak	311
Unsteady separated stagnation-point flow with suction towards a stretching sheet Lok Yian Yian, Syakila Ahmad, and Ioan Pop	317
Utilizing QR decomposition for solving singular fuzzy linear systems M. Nikuie and M. Z. Ahmad	323
A protocol of rope skipping exercise for primary school children: A pilot test A. N. M. Radzi, A. S. Rambely, and K. Chellapan	330
Automated mango fruit assessment using fuzzy logic approach Suzanawati Abu Hasan, Teoh Yeong Kin, Suraiya Sauddin@Sa'duddin, Azlan Abdul Aziz, Mahmod Othman, Ab Razak Mansor, and Vincent Parnabas	335
Computing distance using internal axis-aligned bounding-box for nearly intersected objects Hamzah Asyrani Sulaiman, Abdullah Bade, and Mohd Harun Abdullah	343
Dynamics of two nonlinear oligopoly models Adyda Ibrahim	350

The application of peridynamic method on prediction of viscoelastic materials behaviour N. Nikabdullah, M. A. Azizi, R. Alebrahim, S. S. K. Singh, and Elwaleed A. K.	357
Free convection on heated horizontal circular cylinder in presence of heat generation Najahulfazliah Zainuddin, Ishak Hashim, Muhaimin Ismoen, and Rozaini Roslan	364
Line clipping based on parallelism approach and midpoint intersection Adel Salem Abdullah Hattab and Yuhanis Yusof	371
Stability analysis of the Euler discretization for SIR epidemic model Agus Suryanto	375
Stagnation point flow over a stretching/shrinking cylinder with prescribed surface heat flux Najwa Najib, Norfifah Bachok, and Norihan Md. Arifin	380
Oscillation theory for neutral delay differential equations with variable coefficients Fatima N. Ahmed, Rokiah Rozita Ahmad, Ummul Khair Salma Din, and Mohd Salmi Md Noorani	387
Optimal series solution for mixed convection flow of third grade viscoelastic fluid in a channel with walls transpiration Mohammed Abdulhameed, Ishak Hashim, Habibi Saleh, and Rozaini Roslan	393
Optimization of Spinal Muscular Atrophy subject's muscle activity during gait Gazlia Umat and Azmin Sham Rambely	408
Parametric approach for an absolute value linear fractional programming with interval coefficients in the objective function M. Borza, A. S. Rambely, and M. Saraj	415
Three-dimensional viscous flow over an unsteady permeable stretching/shrinking sheet Mohd Ezad Hafidz Hafidzuddin, Roslinda Nazar, Norihan Md Arifin, and Ioan Pop	422
Surface interpolation using partial differentiation equation with positivity preserving cubic said-ball curves boundary condition Ahmed Kherd, Azizan Saaban, and Noraziah Haji Man	429
Boundary layer flow and heat transfer past a moving plate with suction and injection Anuar Ishak, Roslinda Nazar, and Ioan Pop	435

Mixed convection boundary layer flow over a stretching vertical sheet in a thermally stratified fluid	443
Anuar Ishak, Roslinda Nazar, and Ioan Pop	

ACTUARIAL SCIENCE

A simulation model for the determination of tabarru' rate in a family takaful	449
Hamizun bin Ismail	

Empirical application of normal mixture GARCH and value-at-risk estimation	453
Zetty Ain Kamaruzzaman and Zaidi Isa	

Empirical study on impact of demographic and economic changes on pension cost	460
Shaira Yusof and Rose Irnawaty Ibrahim	

Prediction of interest rate using CKLS model with stochastic parameters	467
Khor Chia Ying and Pooi Ah Hin	

The Lexis plot for run-off non-life insurance companies in United Kingdom	473
Humaida Banu Samsudin	

Static vs stochastic optimization: A case study of FTSE Bursa Malaysia sectorial indices	480
Nur Jumaadzan Zaleha Mamat, Saiful Hafizah Jaaman, and Rokiah@Rozita Ahmad	

Smoothing two-dimensional Malaysian mortality data using <i>P</i>-splines indexed by age and year	487
Halim Shukri Kamaruddin and Noriszura Ismail	

MATHEMATICS EDUCATION

Teaching calculus using module based on cooperative learning strategy	497
Norazman Arbin, Sazelli Abdul Ghani, and Firdaus Mohamad Hamzah	

The relationship between English language learning strategies and gender among pre-university students: An overview of UMS	502
Johannah Jamalul Kiram, Jumat Sulaiman, Suyansah Swanto, and Wardatul Akmam Din	

Blooms' separation of the final exam of Engineering Mathematics II: Item reliability using Rasch measurement model	508
Norain Farhana Ahmad Fuaad, Zulkifli Mohd Nopiah, Norgainy Mohd Tawil, Haliza Othman, Izamarlina Asshaari, Mohd Hanif Osman, and Nur Arzilah Ismail	

OPERATIONAL RESEARCH

A new conjugate gradient method for unconstrained optimization with sufficient descent Nurul Hajar Mohd Yusoff, Mustafa Mamat, Mohd Rivaie, and Ismail Mohd	514
A new modification of Hestenes-Stiefel method with descent properties Norrlaili Shapiee, Mohd Rivaie, Mustafa Mamat, and Ismail Mohd	520
Fletcher Reeves like CG formula approach on Broyden family update A. Z. M. Sofi, M. Mamat, I. Mohd, and M. A. H. Ibrahim	527
Linear versus quadratic portfolio optimization model with transaction cost Norhidayah Bt Ab Razak, Karmila Hanim Kamil, and Siti Masitah Elias	533
Minimizing travel claims cost with minimal-spanning tree model Mohd Helmi Jamalluddin, Mohd Azrul Jaafar, Mohd Iskandar Amran, Mohd Sharizal Ainul, Aqmar Hamid, Zafirah Mohd Mansor, and Zulkifli Mohd Nopiah	541
Multiobjective fuzzy stochastic linear programming problems with inexact probability distribution Abdulqader Othman Hamadameen and Zaitul Marlizawati Zainuddin	546
Operator performance evaluation using multi criteria decision making methods Ruzanita Mat Rani, Wan Rosmanira Ismail, and Siti Fatimah Razali	559
Review of feed forward neural network classification preprocessing techniques Roya Asadi and Sameem Abdul Kareem	567
Solving unconstrained optimization with a new type of conjugate gradient method Syazni Shoid, Mohd Rivaie, Mustafa Mamat, and Ismail Mohd	574
The relationship between budget allocated and budget utilized of faculties in an academic institution Wan Noor Hayatie Wan Abdul Aziz, Rossidah Wan Abdul Aziz, Adibah Shuib, and Nor Faezah Mohamad Razi	580

A model for routing problem in quay management problem Mourad Zirour, Ahmed Oughalime, Choong-Yeun Liong, Wan Rosmanira Ismail, and Khairuddin Omar	587
Heuristic for vehicle routing problem with release and due dates Farhana Johar, Chris Potts, and Julia Bennell	594
PURE MATHEMATICS	
Global optimization using homotopy with 2-step predictor-corrector method Kerk Lee Chang and Rohanin Bt. Ahmad	601
A new concept of similarity measure for IT2FS TOPSIS and its use in decision making Adawiyah Otheman and Lazim Abdullah	608
A new numerical approach for uniquely solvable exterior Riemann-Hilbert problem on region with corners Zamzana Zamzamid, Ali H. M. Murid, and Munira Ismail	615
A quotient space approach to model nanotori and determine their symmetry groups Mark Loyola, Ma. Louise Antonette De Las Peñas, Eko Budi Santoso, and Grace Estrada	620
An extension of first order limit language Muhammad Azrin Ahmad, Nor Haniza Sarmin, Fong Wan Heng, and Yuhani Yusof	627
Automata for subgroups Fong Wan Heng, Gan Yee Siang, Nor Haniza Sarmin, and Sherzod Turaev	632
Automorphism group of nonabelian groups of order p^3 Nor Haniza Sarmin and Yasamin Barakat	640
Bounded harmonic mappings related to starlike functions Durdane Varol, Melike Aydođan, and Yařar Polatođlu	644
Cesáro partial sums of concave univalent functions Ibtisam Aldawish and Maslina Darus	650

Circulant solutions of the matrix equations $AX=C$ and $AXB=C$ Aldous Cesar F. Bueno	657
Crystallographic flat origami with three flat foldability types of the generating unit Eduard C. Taganap, Ma. Louise Antonette N. De Las Peñas, and Teofina A. Rapanut	662
Decomposition theorem in ideal topological spaces W. AL-omeri, Mohd. Salmi Noorani, and A. AL-Omari	668
Fixed point results of T-Kannan contraction on generalized distance in cone metric spaces Zaid Mohammed Fada'il and Abd Ghafur Bin Ahmad	680
Generalized the Pommerenke integral operators Rabha W. Ibrahim and Maslina Darus	684
Homological functor of a torsion free crystallographic group of dimension five with a nonabelian point group Tan Yee Ting, Nor'ashiqin Mohd. Idrus, Rohaidah Masri, Nor Haniza Sarmin, and Hazzirah Izzati Mat Hassim	691
Infective disease processes based on fractional differential equation Rabha W. Ibrahim and Maslina Darus	696
Integral representations of metacyclic groups over a local field Noor Asma'Adny Mohd Adnan, Nor Haniza Sarmin, Nor Muhainiah Mohd Ali, Zainab Yahya, and Hamisan Rahmat	704
Isomorphism classes of 10-dimensional filiform Leibniz algebras Suzila Mohd Kasim, Isamiddin S. Rakhimov, and Sharifah Kartini Said Husain	708
Mappings on multiaspect fuzzy soft classes Nor Hashimah Sulaiman and Daud Mohamad	716
Non-commutative generalized Latin squares of order 5 with certain number of distinct elements H. V. Chen, A. Y. M. Chin, and S. Sharmini	723

On derivations of low dimensional associative dialgebras Mohamed Abubakar, I. S. Rakhimov, and I. M. Rikhsiboev	730
On isomorphism classes of a subclass of filiform leibniz algebras in dimension 10 N. S. Mohamed, S. K. Said Husain, and I. S. Rakhimov	736
On quasi-symmetry of conjugacies between critical circle maps with break points Sokhobiddin Akhatkulov, Mohd Salmi Md Noorani, Akhtam Dzhililov, and Habibulla Akhadkulov	744
On smoothness of conjugations between break equivalent circle maps Habibulla Akhadkulov, Mohd Salmi Md Noorani, and Sokhobiddin Akhatkulov	749
On some homological functors of Bieberbach group of dimension four with dihedral point group of order eight Siti Afiqah Mohammad, Nor Muhainiah Mohd Ali, Nor Haniza Sarmin, Nor'ashiqin Mohd Idrus, and Rohaidah Masri	754
Probabilistic simple splicing systems Mathuri Selvarajoo, Fong Wan Heng, Nor Haniza Sarmin, and Sherzod Turaev	760
q-analogue of Ruscheweyh operator and its applications to certain subclass of uniformly starlike functions Huda Aldweby and Maslina Darus	767
Q-fuzzy soft matrix and its application Fatma Adam and Nasruddin Hassan	772
Quasiconformal harmonic mappings related to Janowski alpha-spirallike functions Melike Aydoğan and Yaşar Polatoğlu	779
A subclasses of harmonic functions defined by multiplier transformation Khalifa Zayid Al-Shaqsi	785
Solvability criteria for cubic equations over Z_2^* Mansoor Saburov and Mohd Ali Khameini Ahmad	792

Solving fuzzy polynomial equation and the dual fuzzy polynomial equation using the ranking method Nurhakimah Ab. Rahman and Lazim Abdullah	798
Some new properties of modified Jacobsthal and Jacobsthal-Lucas numbers Julius Fergy T. Rabago	805
Some properties of (fw) and (fwr)-supplemented module Majid Mohammed Abed, Mohamed Abubakar, and Abd Ghafur Ahmad	819
Study on new class of convex functions Khadeejah Rasheed Alhindi and Maslina Darus	826
The analysis of crystallographic symmetry types in finite groups Atikah Mohd Sani, Nor Haniza Sarmin, Nooraishikin Adam, and Siti Norziahidayu Amzee Zamri	834
An enumeration on the structure of nonrepeated triples in a three-fold triple system Haslinda Ibrahim, Sharmila Karim, and Mohd Saiful Adli Mohamad	844
The generalization of the exterior square of a Bieberbach group Rohaidah Masri, Hazzirah Izzati Mat Hassim, Nor Haniza Sarmin, Nor Muhainiah Mohd Ali, and Nor'ashiqin Mohd Idrus	849
The generative power of weighted one-sided and regular sticker systems Gan Yee Siang, Fong Wan Heng, Nor Haniza Sarmin, and Sherzod Turaev	855
The orbit graph for some finite solvable groups S. M. S. Omer, N. H. Sarmin, and A. Erfanian	863
Tripled fixed point results in cone metric spaces with c-distance Sahar Mohammad Abusalim and Mohd Salmi Md Noorani	870
Watson-Crick Petri net languages with finite sets of final markings Nurhidaya Mohamad Jan, Fong Wan Heng, Nor Haniza Sarmin, and Sherzod Turaev	876

Why even almost perfect number should not be divisible by 3? A non - almost perfect criterion for even positive integers $n \neq 2^k$ John Rafael M. Antalan	881
A class of meromorphically analytic functions related to Cho-Kwon-Srivastava operator with applications to generalized hypergeometric functions F. Ghanim and M. Darus	886
New subclasses of bi-univalent functions involving polylogarithm functions Saibah Siregar and Maslina Darus	893
A study on arithmetical functions and the prime number theorem Yeoh Saw Imm	899
Generalized $\Psi\rho$-closed sets and generalized $\Psi\rho$-open sets in double fuzzy topological spaces Fatimah. M. Mohammed, M. S. M. Noorani, and A. Ghareeb	909
Reliability analysis and prediction of mixed mode load using Markov Chain Model N. Nikabdullah, S. S. K. Singh, R. Alebrahim, M. A. Azizi, Elwaleed A. K, and M. S. M. Noorani	918
QUALITY AND PRODUCTIVITY	
Application of quality measurement tools in determining the quality of drinking water Nora Muda, Norashikin Ramli, and Zainol Mustafa	925
Assessing efficiency and effectiveness of Malaysian Islamic banks: A two stage DEA analysis Norbaizura Kamarudin, Wan Rosmanira Ismail, and Muhammad Azri Mohd	934
Level of understanding of innovation among the Malaysian executives Norkisme Zainal Abidin, Nur Riza Mohd Suradi, Faridatulazna Shahabuddin, Zainol Mustafa, and Wan Rosmanira Ismail	939
Modeling quality attributes and metrics for web service selection Meysam Ahmadi Oskooei, Salwani binti Mohd Daud, and Fang-Fang Chua	945

STATISTICS

A nonlinear model of gold production in Malaysia Norashikin Ramli, Nora Muda, and Mohd Rozi Umor	953
Adaptive and automatic trimming in testing the equality of two group case Suhaida Abdullah, Sharipah Soaad Syed Yahaya, and Abdul Rahman Othman	959
Bivariate Poisson-weighted exponential distribution with applications Hossein Zamani, Pouya Faroughi, and Noriszura Ismail	964
Dependence values of Asia-Pacific stock markets Ruzanna Ab Razak and Noriszura Ismail	969
Development of car theft crime index in peninsular Malaysia Malina Zulkifli, Noriszura Ismail, Ahmad Mahir Razali, and Maznah Mat Kasim	975
Fisher information of the regression parameters using median ranked set sampling Mohammad Al-Rawwash, Mohd Alodat, and Inad Nawajah	983
Gold price effect on stock market: A Markov switching vector error correction approach Phoong Seuk Wai, Mohd Tahir Ismail, and Sek Siok Kun	990
Impact of global financial crisis on stylized facts between energy markets and stock markets Tan Kim Leng, Chin Wen Cheong, and Tan Siow Hooi	994
Lee-Carter state space modeling: Application to the Malaysia mortality data W. H. Wan Zakiyatussariroh, Z. Mohammad Said, and M. R. Norazan	1002
Markov Chain Monte Carlo estimation for Bayesian approach based on right censored data Alomari Mohammed Ahmed	1009
Maximum likelihood estimation of finite mixture model for economic data Seuk-Yen Phoong and Mohd Tahir Ismail	1016

Modified H-statistic with adaptive Winsorized mean in two groups test Kian Wooi Teh, Suhaida Abdullah, Sharipah Soaad Syed Yahaya, and Zahayu Md Yusof	1021
Odd and even sums of generalized Fibonacci numbers by matrix methods C. K. Ho and Chin-Yoon Chong	1026
On the sum of generalized Fibonacci sequence Chin-Yoon Chong and C. K. Ho	1033
Ordinary kriging base on OLS-WLS fitting semivariogram: Case of gold vein precipitation Nur Ali Amri, Abdul Aziz Jemain, and Wan Fuad Wan Hassan	1039
Pedestrian dynamics via Bayesian networks Ibrahim Venkat, Ahamad Tajudin Khader, and K. G. Subramanian	1046
Performance analysis of rotating disc contactor (RDC) column Wan Nurul Aiffah, Siti Aisyah, Nor Fashihah, and Khairil Anuar	1050
Performance of Brownian-motion-generated universal portfolios Choon Peng Tan and Sook Theng Pang	1059
Prediction of daily rainfall Pooi Ah Hin	1066
Premium analysis for copula model: A case study for Malaysian motor insurance claims Yulia Resti, Noriszura Ismail, and Saiful Hafizah Jaaman	1072
Price returns efficiency of the Shanghai A-Shares Wang Jiang Long, Saiful Hafizah Jaaman, and Humaida Banu Samsudin	1078
Robust estimation procedure in panel data model Nurul Sima Mohamad Shariff and Nor Aishah Hamzah	1085
Robust principal component analysis in water quality index development Zalina Mohd Ali, Noor Akma Ibrahim, Kerrie Mengersen, Mahendran Shitan, and Hafizan Juahir	1091
Survival analysis of cancer patients with multiple endpoints using global score test methodology Zakiyah Zain and John Whitehead	1098

The effect of high leverage points on the logistic ridge regression estimator having multicollinearity Syaiba Balqish Ariffin and Habshah Midi	1105
The selection criteria in determining the robustness of <i>t</i>-test Nor Aishah Ahad and Sharipah Soaad Syed Yahaya	1112
The value-at-risk evaluation of Brent's crude oil market Chin Wen Cheong, Zaidi Isa, Khor Chia Ying, and Ng Sew Lai	1118
Universal portfolios generated by Toeplitz matrices Choon Peng Tan, Sin Yen Chu, and Wei Yeing Pan	1126
Comparative assessment of rule-based and Bayes' theorem as inference engines in diagnosing symptoms for Islamic medication expert system H. Daud, R. Razali, T. J. Low, M. Sabdin, and S. Z. Mohd Zafrul	1132
Comparison between empirical mode decomposition and local linear regression in presence of boundaries Abobaker M. Jaber and Mohd Tahir Ismail	1139
Estimation of count data using mixed Poisson, generalized Poisson and finite Poisson mixture regression models Hossein Zamani, Pouya Faroughi, and Noriszura Ismail	1144
The efficiency of reweighted minimum vector variance Hazlina Ali, Sharipah Soaad Syed Yahaya, and Zurni Omar	1151
Testing the equality of students' performance using Alexander-Govern test with adaptive trimmed means Suhaida Abdullah, Sharipah Soaad Syed Yahaya, and Zahayu Md Yusof	1157
The construction of control chart for PM10 functional data Norshahida Shaadan, Abdul Aziz Jemain, and Sayang Mohd Deni	1161

Stochastic growth logistic model with aftereffect for batch fermentation process Norhayati Rosli, Tawfiqullah Ayoubi, Arifah Bahar, Haliza Abdul Rahman, and Madihah Md Salleh	1168
Association between air pollution and hospital admission: Case study at three monitoring stations in Malaysia Marina Zahari, Wan Zawiah Wan Zin@Ibrahim, Noriszura Ismail, and Tan Hui Ni	1178
An efficient automatic firearm identification system Zun Liang Chuan, Choong-Yeun Liong, Abdul Aziz Jemain, and Nor Azura Md. Ghani	1185
Statistical analysis on 1-dimensional and 2-dimensional thermal dissipation for nickel metal hydride battery system Mohammad Firdaus Abu Hashim, Sivakumar Ramakrishnan, and Ahmad Azmin Mohamad	1190
Design of experiment (DOE) study of biodegradable magnesium alloy synthesized by mechanical alloying using fractional factorial design Emee Marina Salleh, Sivakumar Ramakrishnan, and Zuhailawati Hussain	1196
A new approach in measuring graduate employability skills Mohd Hafiz Zakaria, Bidin Yatim, and Suzilah Ismail	1202
Time-series identification of fatigue strain data using decomposition method Z. M. Nopiah, A. Lennie, M. Z. Nuawi, S. Abdullah, A. Z. Nuryazmin, and M. N. Baharin	1209
A preliminary study on drought events in Peninsular Malaysia Wan Zawiah Wan Zin, Siti Aishah Nahrawi, Abdul Aziz Jemain, and Marina Zahari	1217
Diagonally implicit Runge-Kutta method of order four with minimum phase-lag for solving first order linear ODEs Fudziah Ismail and Mohammed M. Salih	1226

Ordinary Kriging base on OLS-WLS Fitting Semivariogram: Case of Gold Vein Precipitation

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Abstract. The comparison of OLS fitting based on exponential semivariogram models, where the range is 811.76 meters, sill is 37.23 (g/t)^2 and the spherical model base, which the sill is 38.44 (g/t)^2 and range is 921.87 meters, used as a basis for OK prediction. Based on these two parameters, sill and range, obtained that result of kriging variances are almost the same, namely 4.96 g/t for the exponential models, and 4.90 g/t for the spherical models.

Keywords: Semivariogram, OLS and WLS fitting, Ordinary Kriging.

INTRODUCTION

In the gold mining region, something difficult to be achieved which is to find the vein gold deposits should be exploited immediately. The main consideration is the economical factor, since (and this is the general character of the mining industry) that the primary gold is almost always located in remote areas, even in places that are not close to the earth's surface. Therefore, we need sampling, (e.g., by drilling at certain points) to obtain presence information. Samples are expected to be important information for observation points around the site, which was not drilled. Here, then predictions should be implemented.

Geostatistic is a method that includes semivariogram and kriging. Semivariogram is a process that must be executed to obtain the semivariogram, which is the function describes the correlation between the values (or sample points) spatially separated. While kriging is a weighted method, which is used to predict the values ([1, 2, 3], and other authors include [4, 5], using a term estimates) at an unknown location by considering the points that have been known to them. As already known the spatial correlation (refere to the values that have been known about), the values in a location that has not been sampled will be predict. This set of values, which became the basis for the determination of reserves.

Reserve calculations in this paper are based on kriging predictions. While the results of kriging predictions are influenced, especially by the sill and range, which is a parameter obtained by fitting the experimental semivariogram (as a discrete function) to the theoretical semivariogram (continuous function). Sill and range, both of which are important parameters to be used as a stepping-stone when it will perform kriging predictions.

Fitting semivariogram, in this paper using two approaches, namely ordinary least squares (OLS) and weighted least squares (WLS), which is chosen semivariogram models with transition, i.e., spherical and exponential models. Spherical model, selected based on consideration of reliability, and is often used in the study of earth science [6]. While the exponential models used as a comparison. Moderate parameters of comparative results have been selected as the basis for the calculation of kriging. Meaning, that the sill value is not too large and the resulting range is not too long.

There are various types of kriging (disjunctive kriging, indicator kriging, multiple indicator kriging, etc.). But in this paper the prediction is using Ordinary Kriging (OK). Therefore the distribution of the data (both physically and disparity of grades) is uneven, then the variography based on robust semivariogram method [7]. Non-performance of the logarithm of the data (although the data are highly skewed), more considerate to determine the effect on the distribution of prediction results, which will be used as a reference to determine the priority of mine exploitation.

MATERIALS AND METHODS

The research material includes of sampling data from exploration drilling results which is done by the company. Geostatistics, here is the method used for handling the data, starting from semivariogram modeling to fitting of theoretical semivariogram model. While kriging is used to predict the data points which are not sampled.

Data and Location

This paper examines the calculation of gold deposits obtained from the weighted averaging of 138 quartz vein drill samples, in Ciurug area have been assayed. Ciurug is an area owned of PT. Aneka Tambang, UBPE Pongkor, Indonesia which is at elevation from 1,110 m to 1,250 m above sea level, and administratively, included the sub-region of Bayah Lebak. Geographically, the area is bounded by latitude of 106°24'00"E - 106°26'00"E and longitude 06°44'00"S - 06°46'00"S. Tectonically, this area is a part of the Indonesian Tertiary magmatic arc, namely the Sunda-Banda arc of Late Miocene – Pliocene age. Regionally, the area was included within the framework of regional geology of Bayah dome, South Banten [14].

High grade gold which is relatively homogeneous, located on the left (or west side) of research area. While residing on the east side, is a group of low-yield spread (also relatively homogeneous).

Experimental Semivariogram

Semivariogram is one step which is performed after statistical analysis of the behavior of data distribution. Normal distribution often used as an option before semivariogram calculation. In a particular case, the omission of the data distribution (without pruning the outliers, for example), can help to understand the possibility of spatial semivariogram behavior anomaly, after reaching sill on the practical range, for example changing in the behavior of rock structures, and also on other geological phenomena [8, 9].

Theoretically, suppose grade of an ore body at point \mathbf{s} (here $\in \mathbb{R}^2$) was observed at certain points of the \mathbf{s}_i ($i=1, \dots, n$) is a realization of a random process $\{Z(\mathbf{s}): \mathbf{s} \in D\}$ of an ore body, while the intrinsic stationarity is determined through a first differences process of $E(Z(\mathbf{s} + \mathbf{h}) - Z(\mathbf{s})) = 0$, then the intrinsic stationarity is defined as [1],

$$\text{var}(Z(\mathbf{s} + \mathbf{h}) - Z(\mathbf{s})) = 2\gamma(\mathbf{h}) = E\{[Z(\mathbf{s} + \mathbf{h}) - Z(\mathbf{s})]^2\}.$$

Thus the semivariogram estimator is,

$$\hat{\gamma}(\mathbf{h}) = \frac{1}{2} \text{mean}\{[Z(\mathbf{s} + \mathbf{h}) - Z(\mathbf{s})]^2\}.$$

If $Z(\mathbf{s}_i)$ is the value of a variable (say, the gold grade) located in \mathbf{s}_i , $Z(\mathbf{s}_j)$ is the value of variable (also grade gold) which is in \mathbf{s}_j , or located at a distance \mathbf{h} from $Z(\mathbf{s}_i)$, while $N(\mathbf{h})$ is the number of pairs of observation points at lag \mathbf{h} , the experimental semivariogram estimator (known as classical semivariogram) is formulated as a function of [10]

$$\hat{\gamma}(\mathbf{h}) = \frac{1}{2|N(\mathbf{h})|} \sum_{i=1}^{N(\mathbf{h})} [Z(\mathbf{s}_i) - Z(\mathbf{s}_j)]^2$$

Functions which is above, assessed by [11] as susceptible to the emergence of atypical data observations, so by transforming Box & Cox (1964) and assumes a principle to normality, he then fine-tune by presenting a more muscular semivariogram estimator, i.e.,

$$\bar{\gamma}(\mathbf{h}) = \left(\frac{1}{2|N(\mathbf{h})|} \sum_{i=1}^{N(\mathbf{h})} [Z(\mathbf{s}_i) - Z(\mathbf{s}_j)]^{1/2} \right)^4 / \left(0.457 + \frac{0.494}{|N(\mathbf{h})|} \right).$$

Fitting Semivariogram

Fitting experimental semivariogram with reference to the theoretical semivariogram is to obtain a trend line. The objective of this fitting is acquisition of sill and range (or other). Various models of the fitting can be carried out, including the eye observations. But this paper presents the automatic fitting with the two least squares methods, namely OLS and WLS where the objective were compared to obtain the most moderate model. In the least squares fitting, the estimated semivariogram constructed by minimizing the sum squared difference of [1, 12]

$$R(\theta) = \sum_{i=1}^k w_i^2 [\hat{\gamma}_z(\mathbf{h}_i) - \gamma_z(\mathbf{h}_i; \theta)]^2 \text{ namely,}$$

$$\min \sum_{i=1}^k w_i^2 [\hat{\gamma}_z(\mathbf{h}_i) - \gamma_z(\mathbf{h}_i; \theta)]^2.$$

Therefore, the weight OLS is $\sum_{i=1}^k w_i^2 = 1$ then the OLS fitted by taking

$$\min \sum_{i=1}^k [\hat{\gamma}_z(\mathbf{h}_i) - \gamma_z(\mathbf{h}_i; \theta)]^2.$$

The WLS weight is $\sum_{i=1}^k w_i^2 = 1 / \text{var}[\hat{\gamma}_z(\mathbf{h}_i)]$ then the fitting is [1, 12]

$$\min \frac{1}{2} \sum_{i=1}^k N(\mathbf{h}_i) \left[\frac{\hat{\gamma}_z(\mathbf{h}_i)}{\gamma_z(\mathbf{h}_i; \theta)} - 1 \right]^2$$

Fitting with the various stages yield a range (especially practical range) and sill. Semivariogram said to achieve of sill γ_∞ , if [1, 13] $\lim_{|\mathbf{h}| \rightarrow \infty} \gamma(|\mathbf{h}|) = \gamma_\infty < \infty$.

It is important to be noted that there is a difference between factor range and practical range (or the range of spatial dependence), which in turn, in the paper is simplified as a range under the symbol a . Range (for exponential) of the lag \mathbf{h} , where $\gamma(\mathbf{h}) = 0.95\gamma(\infty)$ is the distance at which the semivariogram close to 95% of the sill, while the range factor is a condition before reaching the practical range [2].

Theoretical Semivariogram

The semivariogram theoretical (more noticeable in the visual graph), in general will behave on the increase, ranging from the nugget up to a maximum distance of influence or range (a). After that, because of the variability among the points which no longer has a spatial correlation, then the semivariogram curve will be constant. Various types of theoretical semivariogram presented, but often used in mining geology, as general is a transitional form that has [1],

a. Exponential model, $\gamma(\mathbf{h}) = C_0 + C\{1 - \exp(-|\mathbf{h}|/a)\}$

b. Spherical model $\gamma(\mathbf{h}) = \begin{cases} C_0 + C[(3/2)(|\mathbf{h}|/a) - (1/2)(|\mathbf{h}|/a)^3], & 0 < |\mathbf{h}| \leq a \\ C_0 + C, & |\mathbf{h}| \neq a. \end{cases}$

C_0 is nugget effect, C is sill and a is range.

Both theoretical semivariograms are selected for fitting the experimental semivariogram, i.e. using OLS and WLS.

Ordinary Kriging

Kriging, essentially is a weighted averages method of points on a location. This weight is directly related to the semivariogram. In ordinary kriging (OK), the predictive value at unknown point can be observed with similar data in other locations. This prediction was made by modeling the stationary random function of some random variable, namely $Z(\mathbf{s}_1), \dots, Z(\mathbf{s}_n)$.

Suppose a value to be predicted at the point \mathbf{s}_0 is $\hat{Z}(\mathbf{s}_0)$ in which each random variable has the same probability distribution at all locations, while the expectation value of a random variable is $E(Z)$, then [1]

$$\hat{Z}(\mathbf{s}_0) = \sum_{i=1}^n w_i Z(\mathbf{s}_i).$$

$\hat{Z}(\mathbf{s}_0)$ is the predicted value of $Z(\mathbf{s}_i)$ which is based on data observation z_1, \dots, z_n , while w_i ($i = 1, \dots, n$) is the weight of sample points. Prediction error at the point \mathbf{s}_0 obtained by

$$R(\mathbf{s}_0) = \hat{Z}(\mathbf{s}_0) - Z(\mathbf{s}_0)$$

$$E(R(\mathbf{s}_0)) = E(\hat{Z}(\mathbf{s}_0) - Z(\mathbf{s}_0)) = E\left(\sum_{i=1}^n w_i Z(\mathbf{s}_i) - Z(\mathbf{s}_0)\right) = \sum_{i=1}^n w_i E(Z(\mathbf{s}_i)) - E(Z(\mathbf{s}_0)).$$

Because of Z is stationary, then $E(R(\mathbf{s}_0)) = \sum_{i=1}^n w_i E(Z) - E(Z)$. Term of biasness can be achieved if

$$E(R(\mathbf{s}_0)) = 0, \text{ so } \sum_{i=1}^n w_i = 1.$$

The variance error is

$$\tilde{\sigma}_R^2 = \tilde{\sigma}^2 + \sum_{i=1}^n \sum_{j=1}^n w_i w_j \tilde{C}_{ij} - 2 \sum_{i=1}^n w_i \tilde{C}_{i0}.$$

So that $\tilde{\sigma}_R^2$ minimum, the equation must be deducted by Lagrange multiplier, λ and

$$\tilde{\sigma}_R^2 = \tilde{\sigma}^2 + \sum_{i=1}^n \sum_{j=1}^n w_i w_j \tilde{C}_{ij} - 2 \sum_{i=1}^n w_i \tilde{C}_{i0} - 2\lambda \left(\sum_{i=1}^n w_i - 1\right)$$

Minimum error of variance achieved under conditions where the first partial derivatives of the weights equated to zero,

$$\frac{\partial \tilde{\sigma}_R^2}{\partial w_i} = 0 \text{ so that } \sum_{j=1}^n w_j \tilde{C}_{ij} + \lambda = \tilde{C}_{i0}, \forall i = 1, \dots, n.$$

Boundary condition, generated by searching a first partial derivative of the equation with a Lagrange multiplier and equating to zero

$$\frac{\partial \tilde{\sigma}_R^2}{\partial \lambda} = 0 \text{ then } \sum_{i=1}^n w_i = 1.$$

Equation system of OK can be presented in matrix form as follows,

$$\underbrace{\begin{bmatrix} \tilde{C}_{11} & \dots & \tilde{C}_{1n} & 1 \\ \vdots & \ddots & \vdots & \vdots \\ \tilde{C}_{n1} & \dots & \tilde{C}_{nn} & 1 \\ 1 & \dots & 1 & 0 \end{bmatrix}}_{(n+1) \times (n+1)} \underbrace{\begin{bmatrix} w_1 \\ \vdots \\ w_n \\ \lambda \end{bmatrix}}_{(n+1) \times 1} = \underbrace{\begin{bmatrix} \tilde{C}_{10} \\ \vdots \\ \tilde{C}_{n0} \\ 1 \end{bmatrix}}_{(n+1) \times 1}$$

Within a vector form can be written as follows, $C\vec{W} = \vec{D}$. C is the covariance matrix of the observed data on the location, and usually a positive definite matrix. To gain weight \vec{W} , on the left-hand side of the equation is multiplied by the matrix C^{-1} , so that $\vec{W} = C^{-1}\vec{D}$.

DISCUSSION

The discussion includes the execution of semivariogram (experimental and theoretical) to ordinary kriging predictions which are based on a grid block size of 5×5. Fitting comparison is done using OLS and WLS method, based on the exponential and spherical models.

Semivariogram

Semivariogram is a function (and tool) that are used to describe or determine the variability (level of similarity) of regionalized data separated by \mathbf{h} , up to a certain distance (called, the range of influence) in which the data is no longer spatially correlated. Although after that, anomaly events can occur where semivariogram fluctuated, (can continue ascending, or even a drastic decline).

Semivariogram calculation, in this case, based on the assumption of omnidirectional, meaning, that the semivariogram being equal, for a variety of directions. Each lag distance is 100 meters, which is the maximum distance of 1500 meters (longitude axis is East-West). Longitude starts from 7435.47 to 9069.75, while the latitude is 343.55 to 752.08, and the whole area is 545.606 m². Execution of the data calculation (i.e. semivariogram and kriging) are using the open source program, R package [15].

Although the distribution is skewed (skewness=2.499) so that its distribution is still awake, then the experimental semivariogram calculation is using robust models (data need not be transferred to a logarithmic form). **TABLE 1** presents the values of the semivariogram lag distances (h), regularly every 100 m with abscissa between 7500 to 9000. Therefore, there are 138 data, so, in the semivariogram calculation, there are $n \times (n-1)/2$ or $(138 \times 137)/2 = 9543$ unique pairs of observations.

Maximum pair of points (named, pairs) is 1589, while the minimum is 2. Disparities (colored very sharp) occurs due to the left (Southwest, relatively) is a rich zone (≥ 15 g/t). More towards the east, where the number of samples to be less and less, grade is being smaller, and this is called as a poor zone (≤ 1.5 g/t).

In general, it appears that the trip of robust semivariogram (or, modulus) is fluctuating, though at a certain lag, it was tended to increase. Rise significantly, occurs on the ninth lag, which is at a distance of 900 where the increase is almost double from 36.83 towards 63.73, both in $(g/t)^2$ Au². After that, is decreased again until the eleventh lag.

TABLE (1). Omnidirectional Robust Semivariogram

Lag Distance (m)	Semivariogram ^{*)}	Pairs
100	17.51	1589
200	21.48	1540
300	19.88	1268
400	21.06	1114
500	29.51	981
600	30.62	840
700	23.88	629
800	36.83	514
900	63.73	370
1000	51.17	280
1100	44.17	170
1200	55.83	94
1300	44.26	39
1400	15.87	23
1500	1.51	2

^{*)}in $(g/t)^2$ Au.

Fitting Semivariogram

Fitting the experimental semivariogram, especially performed to obtain sill and range (*a*), which on inquiry was conducted against the exponential and spherical models. Based on **TABLE 2**, robust semivariogram fitting OLS models yield a smaller sill (or might say better) than the WLS. Distance between the influence of sample points is also larger (or longer) than the range of the sample points on the overall classical semivariogram.

TABLE (2). Robust Semivariogram Parameters

	OLS	WLS
<i>Exponential</i>		
Sill $(g/t)^2$	37.23	50.72
Range (m)	811.76	1608.82
<i>Spherical</i>		
Sill $(g/t)^2$	38.44	47.80
Range (m)	921.87	1139.96

The range with WLS fitting for exponential models is 1608.82 m, almost twice of range using OLS, which is 811.76 m (797 m differences). This distance is too far from the maximum distance (1500 m). In the spherical case, the difference between the OLS model of sill and WLS is only 9.36 $(g/t)^2$, whereas the difference in range is 218 m.

Based on these results, obtained the general form of the two models OLS fitting estimation of semivariogram basis, which exponential models is

$$\hat{\gamma}(\mathbf{h}) = 37.23\{1 - \exp(-|\mathbf{h}|/811.76)\}.$$

As for the spherical models is

$$\hat{\gamma}(\mathbf{h}) = \begin{cases} 38.44[(1.5)(|\mathbf{h}|/921.87) - (0.5)(|\mathbf{h}|/921.87)^3], & 0 < |\mathbf{h}| \leq 921.87 \\ 38.44, & |\mathbf{h}| \neq 921.87. \end{cases}$$

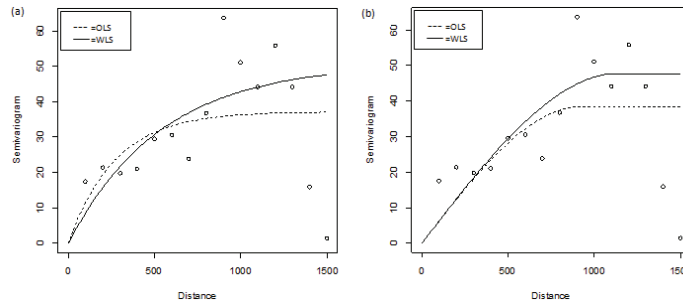


FIGURE 1. Fitting base on OLS and WLS Robust Semivariogram of (a) Exponential; and (b) Spherical

Ordinary Kriging

Journal (1983) states, that the effectiveness indication of the spatial data completion using kriging method can be carried out by looking at two things, namely the skewness and the coefficient of variation (CV). The data, which have a very uneven distribution or skewed (where the skewness is greater than 1.5), more suited solved by kriging method. More specifically, linear kriging will be effective to solve the data that has CV is less than one. In this case, the value of CV is 0.883 ($CV < 1$), while the skewness is 2.499 (> 1.5). Meaning, these data will be effectively solved by the kriging method, in particularly is a linear kriging (OK).

The prediction of OK performed on the area of 375,000 m², where the size of each block is (5×5) m², using two models, exponential and spherical. Using OLS fitting exponential models, the value of sill (C) is 37.23 (g/t)², at a distance of influence (a) along the 811.76 m. Whereas at the spherical models, sill value (C) is 38.44 (g/t)² which the distance between the influence of spatial points (a) is 921.87 m. Based on FIGURE 2, it appears that the rich zone (≥ 15 g/t) spreads near longitude of 7650-7850, with a yellow to green colored (bright), is in a position of relatively to the Southwest area of about 2,500 m². Poor zone almost spread in most of the middle to eastern sampling locations, light blue to purple (dark).

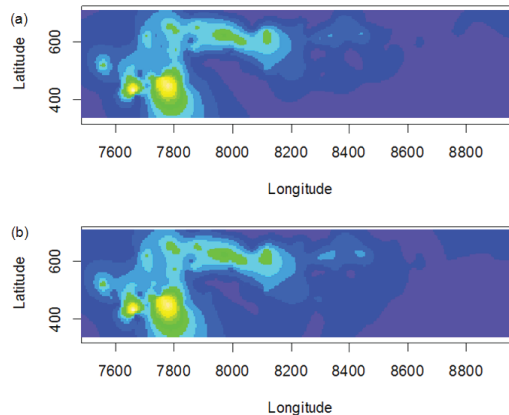


FIGURE 2. Ordinary Kriging Estimation base on Fitting OLS model (a) Exponential ($C=37.23$ & $a=811.76$) dan; (b) Spherical ($C=38.44$ & $a=921.87$)

Fitting of spherical semivariogram (FIGURE 2 b), produces the distribution of grade, medium to high which is slightly wider than the exponential models. In contrast, the exponential models produces of low grade distribution

(purple color) and it is wider, almost breaking into high grade (**FIGURE 2 a**). Mean prediction, both as **TABLE 3** is almost same, namely 4.96 g/t for exponential and 4.90 g/t for spherical. Nonetheless, the OK standard deviation (square of variance) of the prediction results (exponential models) is smaller (ie, 1.75 g/t) than the OK standard deviation of spherical models, namely 2.08 g/t.

TABLE (3). Ordinary Kriging Parameters (g/t)

Kriging Parameters	Exponential	Spherical
Mean Estimation	4.96	4.90
Median Estimation	3.29	3.30
3 rd Quantile Estimation	6.83	6.82
Kriging Std. Dev.	1.75	2.08

CONCLUSION

Based on the calculation of robust experimental semivariogram (both theoretical semivariogram fitting exponential models and models of spherical) obtained information that OLS fitting yields range is smaller than the WLS fitting. Even on exponential models, WLS fitting yields range of 1608 meters, and is beyond of maximum distance (i.e., 1500 m). Meaning, spatial effect between sample points is much greater than half of the longitude length, which is a gathering sites of grade rich.

OK prediction produces mean, median and 3rd quantile relatively the same, although the kriging variance (expressed in standard deviation) are slightly different. Predictions also show, that the rich zone located in the Southwest region, relatively.

The information of data distributions in the mining industry, serve as a very useful reference in calculating of reserves (or resources), and also, in determining the priority scale of mining operation (i.e., in mining exploitation).

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