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Determination of Initial Lag and Maximum Lag Distance of Semivariogram on Reserves Calculation of Gold Vein

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Abstract

The reserves calculation is one of the early stages before the mining operation, which begins with the samples prediction. Ordinary kriging is a part of the kriging method that works with semivariogram parameters. The calculation of these parameters based on several things, including the determination of the lag distance and the fitting model. This paper is based on the determination of the maximum lag distance of 500 and 800 where the lag distance for both, respectively 15, 25 and 35 with WLS and OLS fitting models. The six combinations can be concluded that increasing of maximum lag distance adds to a large of sill and extend the range. OK prediction shows that the largest gold grade mean (5.032) occurs in the kriging using WLS fitting models, in which the maximum lag distance semivariogram is 500 and the distance of each lag is 35.

Keywords: Semivariogram, Lag, Gold.

1. Introduction

Determination of lag distance on drilling results data with randomly spaced (Popoff, 1966) is not as easy as the data generated by systematic drilling in which the sample supports are regularly sized spaces. The lag distance determination is an important first stage, which forms the cornerstone in the calculation of semivariogram parameters. In geostatistics, semivariogram parameters in particularly nugget, range and sill are largely determined by fitting of the experimental semivariogram.

2. Method

This paper presents three things, first is looking for the experimental semivariogram by setting of the initial lag distance and the different of maximum lag distance. Determination of the lag distance is one of the determinants obtained semivariogram parameters, i.e. as values on the basis of execution of kriging predictions. Having obtained the semivariogram in which result of the experiment is still in form of the discrete values, second step is fitting them with the theoretical semivariogram. Fitting is, essentially translation of discrete curves into a continuous form, which is facilitated by a mathematical equation, in this study is the spherical equation. Third, apply it in to the kriging prediction. Kriging works with major basic components derived from fitting semivariogram i.e. nugget, sill and range. The execution of programs on this article using the R-package, especially the library(geoR) [Bivand, et al. 2008].

Fitting as a means for translating the behavior semivariogram, in this study using the weighted least squares (WLS) and ordinary least squares (OLS) based on the theoretical of spherical semivariogram. The results of parameters are used as the basis for calculating the ordinary kriging prediction. Experimental semivariogram estimator, known as classical semivariogram presented (Matheron, 1963),

$$\hat{\gamma}(\mathbf{h}) = \frac{1}{2|N(\mathbf{h})|}\sum_{i=1}^{N(\mathbf{h})}[Z(\mathbf{s}_i) - Z(\mathbf{s}_j)]^2$$

where $\mathbf{h} = \mathbf{s}_i - \mathbf{s}_j$. *This experimental semivariogram known as the classical or empirical semivariogram model. In consideration of normality and the fourth root of chi-square, Cressie and Hawkins (1980) then refine it through the transformation of*

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Box & Cox (1964) which is becomes as robust (here used),

$$\bar{\gamma}(\mathbf{h}) = \left(\frac{1}{2|N(\mathbf{h})|} \sum_{i=1}^{N(\mathbf{h})} [Z(\mathbf{s}_i) - Z(\mathbf{s}_j)]^{1/2} \right)^4 / \left(0.457 + \frac{0.494}{|N(\mathbf{h})|} \right).$$

Experimental semivariogram omni-directional (Kushavand, 2007) which is still in a discrete expression needs to be fitted in a continuous form. Two continuous semivariogram theoretical used in this study is a form of spherical. Spherical models formulated as common use (Cressie, 1993, Zhang, et al. 2009).

$$\gamma(\mathbf{h};\theta) = \begin{cases} 0, & \mathbf{h} = 0 \\ C_0 + C\{(3/2)(|\mathbf{h}|/a) \quad (1/2)(|\mathbf{h}|/a)^3, & 0 < |\mathbf{h}| \le a \\ C_0 + C, & |\mathbf{h}| \ge a. \end{cases}$$

In this case, the tangent at the origin point cut the sill at the point where the abscissa position is 2a/3. Least squares fitting estimation is done by minimizing the sum of squared difference R(θ),

$$\min \sum_{\mathbf{i}=1}^{\mathbf{k}} \mathbf{w}_{\mathbf{i}}^2 [\hat{\gamma}_z(\mathbf{h}_i) - \hat{\gamma}_z(\mathbf{h}_i;\boldsymbol{\theta})]^2$$ *where,*

$$R(\theta) = \sum_{i=1}^{k} w_i^2 [\hat{\gamma}_z(\mathbf{h}_i) - \hat{\gamma}_z(\mathbf{h}_i;\theta)]^2.$$

OLS and WLS are two of the least squares fitting models used in the study. It is assumed that the sum of squares of weights, $\sum_{i=1}^{k} w_i^2 = 1$*, so the OLS estimates is*

$$\min = \sum_{\mathbf{i}=1}^{\mathbf{k}} [\hat{\gamma}_z(\mathbf{h}_i) - \gamma_z(\mathbf{h}_i;\boldsymbol{\theta})]^2.$$

Because of $w_i^2 = 1/Var[\hat{\gamma}_z(\mathbf{h}_i)] \approx 1/2 \frac{\gamma(\mathbf{h}_i,\theta)^2}{N(\mathbf{h}_i)}$

WLS estimation to be

$$\min \frac{1}{2} \sum_{i=1}^{k} N(\mathbf{h}_i) \left[\frac{\hat{\gamma}_z(\mathbf{h}_i)}{\gamma_z(\mathbf{h}_i;\theta)} - 1 \right]^2.$$

*Spatial sample is a representation of data population which is expected to represent other un-sampled data in the region. If Z(**s**$_0$) is an un-sampled point value (gold-grade) at the location **s**$_0$ then the prediction values, i.e.* $\hat{Z}(\mathbf{s}_0)$ *according to kriging obtained by the formula*

$$\hat{Z}(\mathbf{s}_0) = \sum_{i=1}^{n} w_i Z(\mathbf{s}_i).$$

Here, w_i *is the i-th weight w of **s**$_0$. Un-biased conditions occur if* $\sum_{\mathbf{i}=1}^{\mathbf{n}} \mathbf{w}_{\mathbf{i}} = 1$*, so for the average becomes*

$$E[\hat{Z}(\mathbf{s}_0)] = E\left[\sum_{i=1}^{n} w_i Z(\mathbf{s}_i)\right] = \sum_{i=1}^{n} w_i E[Z(\mathbf{s}_i)] = \mu \sum_{i=1}^{n} w_i.$$

Weighting also aims to look for un-biased variance and a minimum estimation. Un-biased occurs when the mean estimation is equal to the mean of the real value. Minimum estimates shown by

$$\min \sigma_e^2 = E[Z(\mathbf{s}_0) - \hat{Z}(\mathbf{s}_0)] = E[Z(\mathbf{s}_0) - \sum_{i=1}^{n} w_i Z(\mathbf{s}_i)].$$

While the variance estimation of ordinary kriging obtained by

$$\sigma_e^2 = 2\sum_{i=1}^{n} w_i \gamma(\mathbf{s}_0 - \mathbf{s}_i) - \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \gamma(\mathbf{s}_i - \mathbf{s}_j).$$

3. Result and Discussion

Semivariogram

The maximum lag distance which is taken is 500 and 800. The consideration is that the distance of 500 one-third of the location of the data landscape. While 800 is selected as a distance greater than half the span (750). The length of the data span is ± 1500 whose position had been the same as the direction vein. Lag distance used for each of the two maximum lag distance is 15, 25 and 35. The result of the calculations is in Table 1.

Table 1. Experimental semivariogram data with maximum lag distance of 500 and 800

Lag	*Lag distance=15*			*Lag distance=25*			*Lag distance=35*		
	Lag dist.	*Pairs*	$\hat{\gamma}(\mathbf{h})$	*Lag dist.*	*Pairs*	$\hat{\gamma}(\mathbf{h})$	*Lag dist.*	*Pairs*	$\hat{\gamma}(\mathbf{h})$
1	*15*	*33*	*4.343*	*25*	*85*	*4.826*	*35*	*216*	*6.624*
2	*30*	*52*	*5.132*	*50*	*204*	*8.458*	*70*	*316*	*8.888*
3	*45*	*131*	*7.791*	*75*	*243*	*8.658*	*105*	*421*	*9.349*
4	*60*	*137*	*8.474*	*100*	*319*	*9.586*	*140*	*493*	*12.075*
5	*75*	*113*	*8.894*	*125*	*315*	*11.315*	*175*	*464*	*12.231*
6	*90*	*184*	*9.938*	*150*	*332*	*11.842*	*210*	*451*	*12.199*
7	*105*	*201*	*9.314*	*175*	*355*	*12.014*	*245*	*415*	*13.133*
8	*120*	*175*	*8.905*	*200*	*321*	*12.406*	*280*	*420*	*12.961*

9	*135*	*207*	*13.204*	*225*	*319*	*12.835*	*315*	*354*	*12.520*
10	*150*	*213*	*11.922*	*250*	*283*	*12.793*	*350*	*375*	*11.059*
11	*165*	*178*	*11.163*	*275*	*306*	*12.933*	*385*	*338*	*12.845*
12	*180*	*229*	*12.851*	*300*	*248*	*13.525*	*420*	*315*	*13.694*
13	*195*	*197*	*12.278*	*325*	*281*	*11.630*	*455*	*323*	*14.414*
14	*210*	*194*	*13.582*	*350*	*265*	*10.939*	*490*	*292*	*16.777*
15	*225*	*187*	*11.916*	*375*	*236*	*12.926*	*525*	*289*	*17.882*
16	*240*	*173*	*12.486*	*400*	*256*	*13.895*	*560*	*274*	*16.279*
17	*255*	*172*	*13.112*	*425*	*210*	*12.342*	*595*	*246*	*15.943*
18	*270*	*203*	*12.889*	*450*	*248*	*13.626*	*630*	*252*	*13.557*
19	*285*	*162*	*12.677*	*475*	*207*	*16.836*	*665*	*214*	*15.694*
20	*300*	*141*	*14.172*	*500*	*97*	*19.987*	*700*	*206*	*11.699*

Description: At lag=35 and maximum lag distance=500 the semivariogram values just ended up to lag -14.

Unless the lag=35 and maximum lag distance is 500, there are just 14 semivariogram values. To simplify the presentation, Table 1 is only displayed 20 items experimental semivariogram.

Semivariogram Parameter

*The three main parameters to be searched are nugget, sill and range. Sill is the visualization of spatial variance data; range is the distance in which no spatial mainstreaming between the points. While the nugget is a condition in which the extrapolation curve leading to **h**=0 do not give semivariogram at that point is zero [$\gamma(0)\neq 0$], but $\gamma(0)=C_0$. Knowledge of nugget effect (C_0) in the practice of mining geology can provide important information supply to the phenomenon being investigated, so that will have implications for the estimation (or prediction) resource ore (even the reserves) and in selecting of mining methods (Carrasco, 2010). Nugget effect also indicates the enrichment zone of mineralization in the fracture, as the case of deposition of nickel laterite [Heriawan, et al. (2009)].*

Table 2 presents the result calculation of the maximum lag distance of 500 fitted in two models, WLS and OLS, which is each a lag of 15, 25 and 35. While Table 3 is the result of calculation which is based on the maximum lag distance of 800, i.e. a number greater than 750 (half-span). In the OLS model with a maximum lag distance of 500 obtained that the nugget effect is same or smaller than the WLS models. The range size of the OLS model also smaller than WLS models, except for lag=15 which is slightly larger, i.e. 167.982 (Table 2). In contrast, the values of sill in the OLS model, precisely greater than the WLS models.

Table 2. semivariogram parameters of maximum lag distance 500

Semivariogram parameter	*Spherical model*					
	WLS of lag distance			*OLS of lag distance*		
	15	*25*	*35*	*15*	*25*	*35*
Nugget	*2.65*	*2.00*	*3.5*	*2.65*	*1.60*	*2.9*
Sill	*13.002*	*12.960*	*13.003*	*13.139*	*13.350*	*13.232*
Range	*167.981*	*163.909*	*168.476*	*167.982*	*162.139*	*162.866*

In the semivariogram where the maximum lag distance is 800 (Table 3), it appears that there are same value of nugget (WLS and OLS) in each lag distance. Here, the value of sill with the OLS model is also smaller than WLS model, except at a distance of lag 35 (i.e. amount to 14.397). In contrast, occurs in the range calculation where the OLS model is larger than the WLS model (except at lag distance of 35, i.e. 198.421.

Table 3. semivariogram parameters of maximum lag distance of 800

Semivariogram parameter	*Spherical*					
	WLS of lag distance			*OLS of lag distance*		
	15	*25*	*35*	*15*	*25*	*35*
Nugget	*2.25*	*2.45*	*3.75*	*2.45*	*2.45*	*3.75*
Sill	*14.088*	*14.050*	*13.995*	*14.546*	*14.587*	*14.397*
Range	*203.728*	*199.342*	*199.574*	*206.004*	*203.257*	*198.421*

In general obtained information, the enlargement value of maximum lag distance also caused more value of the sill and range. If viewed from the smallest nugget effect in a maximum lag distance of 500 (i.e. 1.6 for OLS model), the determination of the best value from the initial lag is 25. This applies to both WLS and OLS models. However, if the maximum lag distance is increased to 800, the selection of the best lag distance is equal to 15 (WLS), which the nugget is 2.25.

Kriging of Maximum Lag Distance=500

In the gold mining upstream industry, kriging become one of the decisive operations of enterprises, especially resource and reserves determining. Table 4 represents the results of ordinary kriging prediction at a maximum lag distance of 500. At this lag distance, the largest mean value resulting execution is 5032 and it obtained from the block kriging in dimension of 20×20, and is the result of fitting WLS spherical semivariogram models where the lag distance is same, namely 35. However, if the consideration is the average of the smallest variance (i.e. 7.243) the best predictions generated by WLS semivariogram base of a lag distance=25 and the maximum lag distance of 500, where the average grade is 4,574.

Table 4. Prediction and variance of kriging based on spherical semivariogram with maximum lag distance of 500

Category		Min	1st Qu.	Median	Mean	3rd Qu.	Max
	Data	0.450	2.650	5.275	5.823	8.362	14.800
WLS	Prediction	1.348	3.527	4.503	4.971	5.781	12.870
(15)	Variance	4.113	6.484	9.502	10.220	14.100	16.210
OLS	Prediction	1.344	3.525	4.501	4.970	5.783	12.880
(15)	Variance	4.119	6.517	9.565	10.290	14.220	16.360
WLS	Prediction	1.317	2.644	3.615	4.574	5.988	12.810
(25)	Variance	3.048	4.589	6.458	7.243	9.545	15.150
OLS	Prediction	1.197	3.460	4.530	4.986	5.859	13.220
(25)	Variance	2.664	4.948	7.706	8.376	12.030	13.820
WLS	Prediction	1.562	3.657	4.612	5.032	5.785	12.320
(35)	Variance	4.952	6.740	8.916	9.328	12.070	13.440
OLS	Prediction	1.451	3.627	4.648	5.030	5.765	12.600
(35)	Variance	4.319	6.244	8.710	9.173	12.230	13.670

A comparison of the distribution of the two prediction results shown in Figure 6, where (a) is a prediction with the smallest mean (i.e. 4574) using the WLS spherical fitting model at a lag distance of 25 and the maximum lag distance of 500. While (b) is the distribution generates the largest mean of gold grade (i.e. 5.032) using the WLS spherical models fitting in which the lag distance of 35 and maximum lag distance of 500.

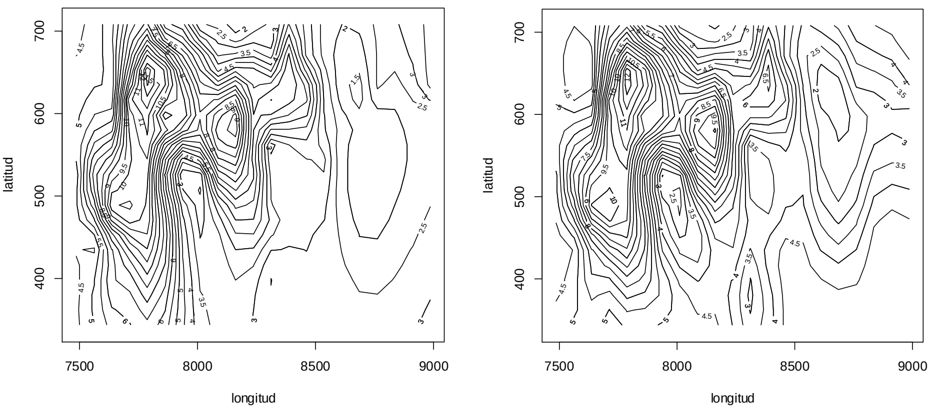
Kriging of Maximum Lag Distance=800

Tables 5. Prediction and variance of kriging based on spherical semivariogram with maximum lag distance of 800

Category		**Min**	**1st Qu.**	**Median**	**Mean**	**3rd Qu.**	**Max**
	Data	0.450	2.650	5.275	5.823	8.362	14.800
WLS	Prediction	1.386	3.178	4.133	4.782	5.945	12.830
(15)	Variance	3.430	5.183	7.538	8.422	11.660	14.740
OLS	Prediction	1.407	3.190	4.153	4.779	5.955	12.770
(15)	Variance	3.696	5.456	7.856	8.751	12.020	15.220
WLS	Prediction	1.414	3.229	4.213	4.809	5.892	12.740
(25)	Variance	3.692	5.439	7.811	8.629	11.869	14.680
OLS	Prediction	1.404	3.201	4.147	4.789	5.960	12.780

(25)	*Variance*	*3.705*	*5.499*	*7.933*	*8.819*	*12.150*	*15.260*
WLS	*Prediction*	*1.585*	*3.371*	*4.326*	*4.867*	*5.919*	*12.260*
(35)	*Variance*	*5.173*	*6.809*	*8.908*	*9.519*	*12.310*	*14.570*
OLS	*Prediction*	*1.572*	*3.360*	*4.315*	*4.867*	*5.894*	*12.290*
(35)	*Variance*	*5.207*	*6.902*	*9.100*	*9.737*	*12.660*	*14.990*

In Table 5, in which the determination of maximum lag distance is 800, there are two result calculations of mean of the biggest grade based on OK prediction (i.e. 4.867) which is at a lag distance of 35, results of the fitting OLS and WLS models. Both are based on spherical models. However, if the consideration is the average of smallest kriging variance (i.e. 8.422), the best results are semivariogran based on fitting results of OLS spherical model, namely the average grade of 4782.



(a) *(b)*

Figure 1. Comparison of the result of grade prediction: (a) the smallest, i.e. 4.574 (WLS 25_500); (b) the largest, 5.032 (WLS 35_500)

4. Conclusion

Some issues related to this research (where the data is randomly presented) are that,

1. *The enlargement of maximum lag distance will further enlarge the sill and extend the range.*
2. *The smallest nugget effect (i.e. 1.6) occur in cases where the maximum of lag distance is 500 (one-third of the maximum span) and initial lag of 25, and this happened to the semivariogram result of OLS fitting. The mean of kriging prediction is 4.486 with a variance of 8376.*
3. *The biggest average of gold grade (i.e. 5032) occurs in kriging prediction that is uses of WLS model fitting base, where the maximum lag distance of semivariogram is 500 and the distance of each lag is 35. In this condition, the nugget effect is 3.5 and range is 168.476. And the largest value precisely. It may well be, that this nugget effect is the real condition which is related to the enrichment of gold ore mineralization in the fracture zone in one of the region. Smallest grade (i.e. 4.575) occurred in the kriging prediction based on WLS model in which the maximum lag distance of 500 and the distance lag is 25. Contour visual comparison of the two models shown in Figure 1.*
4. *While the range of 168.476 for Ciurug gold vein case is the ideal value, became the basis of the (ordinary) kriging prediction.*

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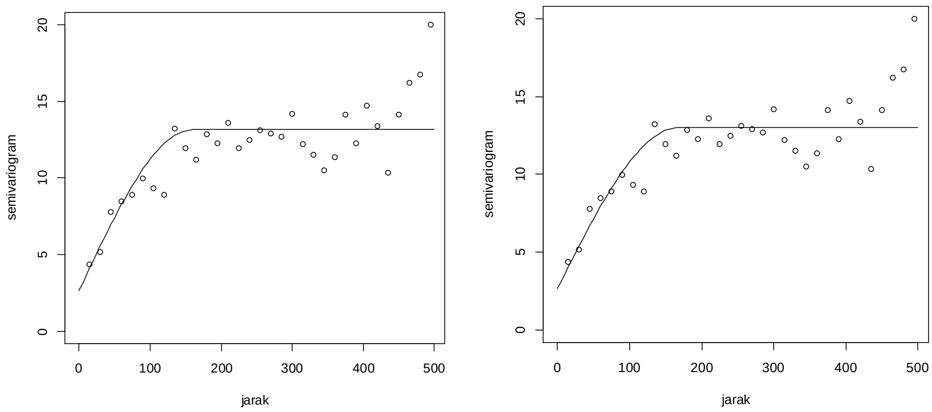
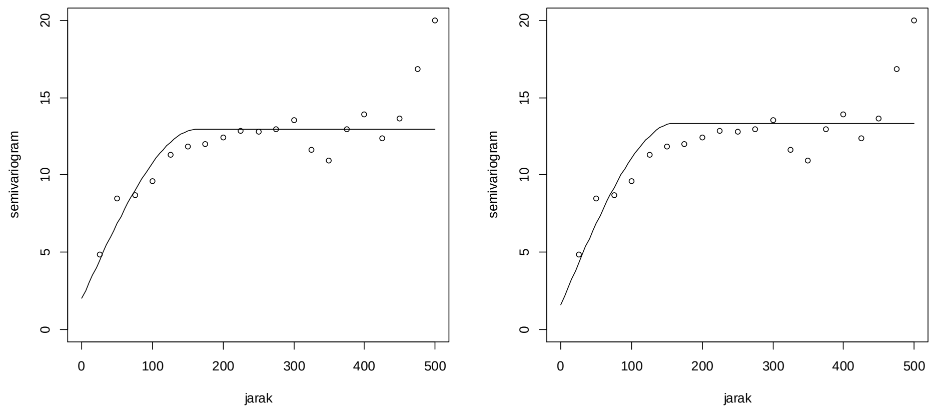
Lampiran - Lampiran

Grafik Hasil Fitting Semivariogram Klasikal Model Sferis dan Data



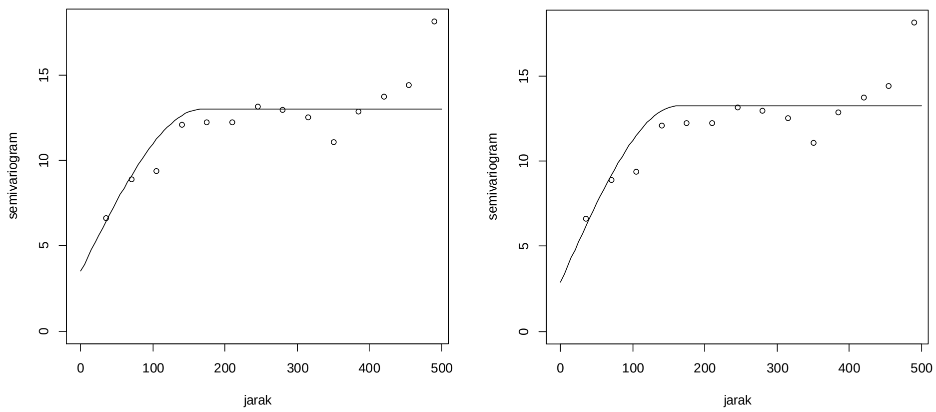
(a) *(b)*

Grafik 1. Visual lag=15, jarak maksimum lag=500 (a) WLS sferis (b) OLS sferis



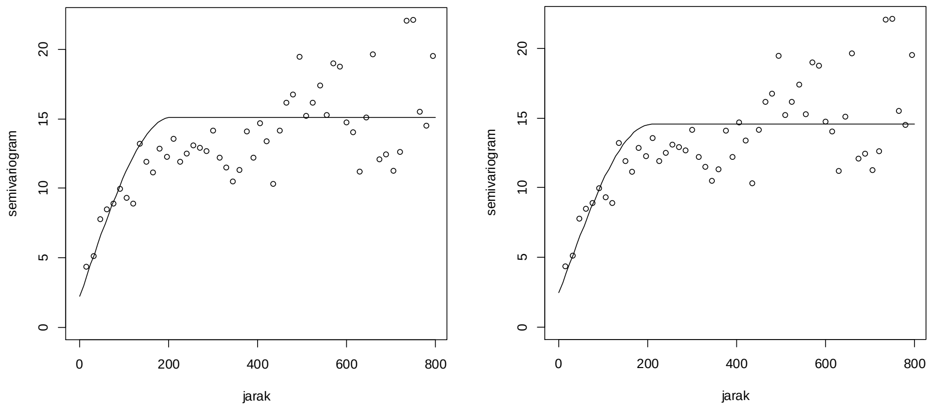
(b) *(b)*

Grafik 2. Visual lag=25 jarak maksimum lag=500 (a) WLS sferis (b) OLS sferis

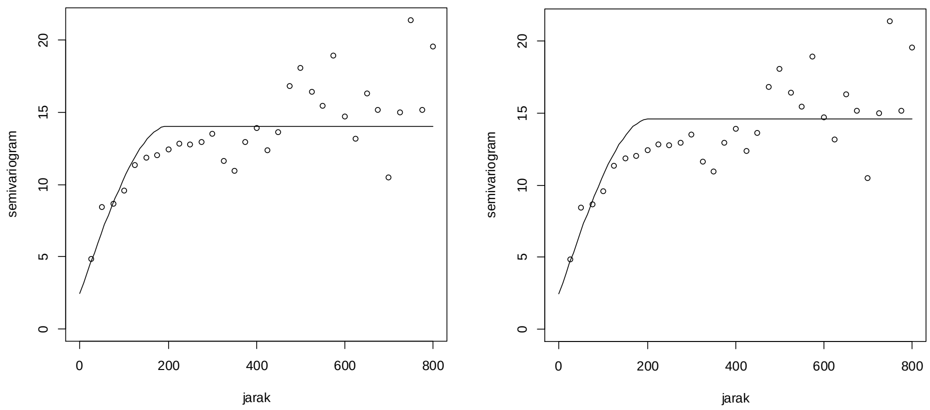


(c) *(b)*

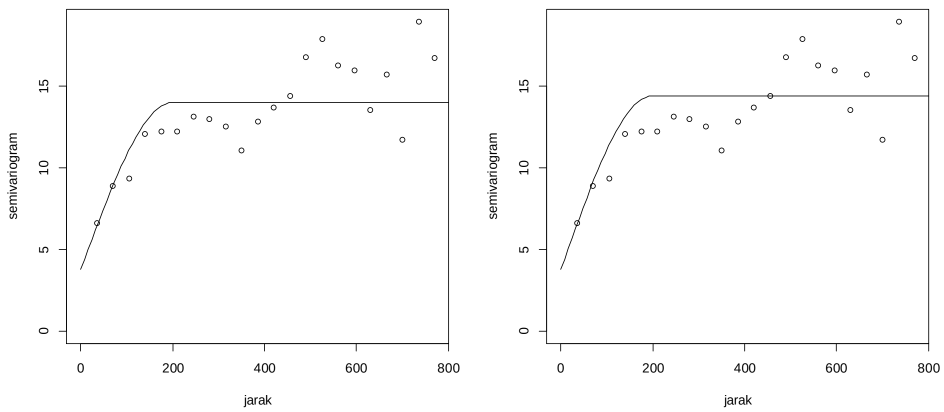
Grafik 3. Visual lag=35 jarak maksimum lag=500 (a) WLS sferis (b) OLS sferis



Grafik 4. Visual lag=15, jarak maksimum lag=800 (a) WLS sferis (b) OLS sferis

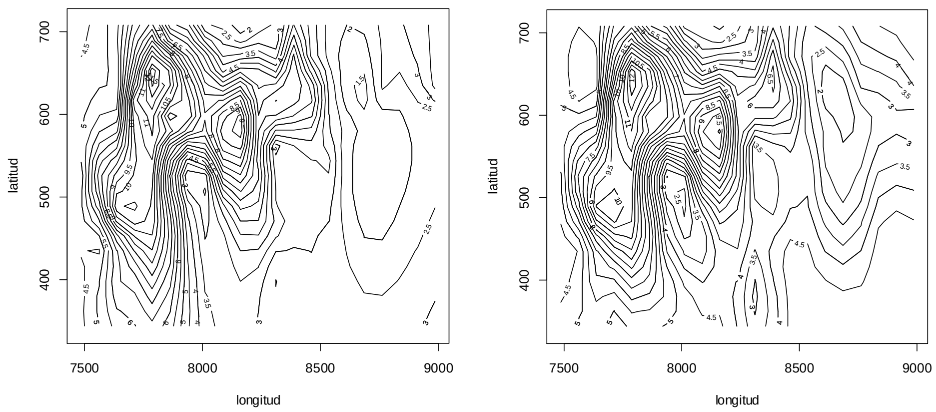


Grafik 5. Visual lag=25 jarak maksimum lag=800 (a) WLS sferis (b) OLS sferis



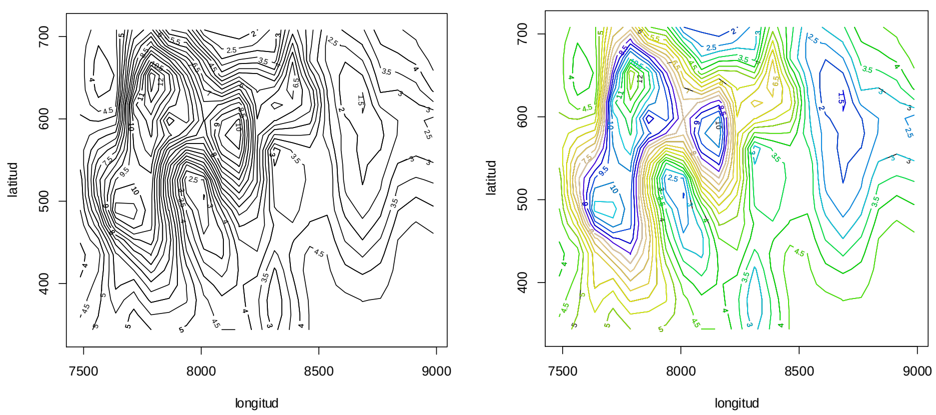
Grafik 6. Visual lag=35 jarak maksimum lag=800 (a) WLS sferis (b) OLS sferis

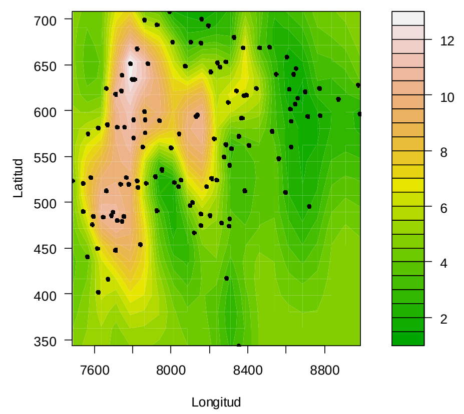
Komparasi Hasil Prediksi Gred



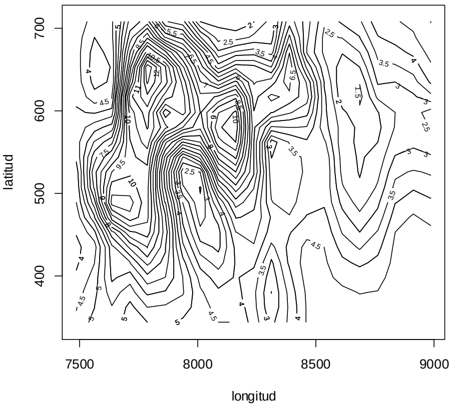
(b) *(b)*

Figure 1. Comparison of the results of grade prediction: (a) the smallest, i.e. 4.574 (WLS 25_500)
(b) the largest, 5.032 (WLS 35_500)





KWLSS 15 500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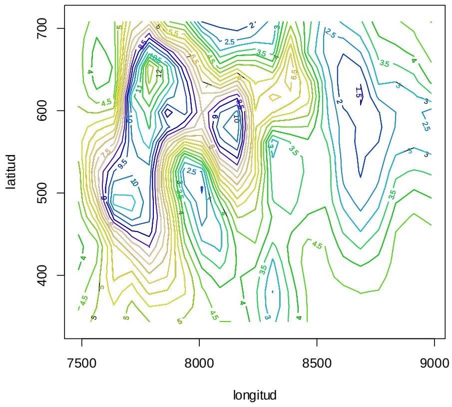
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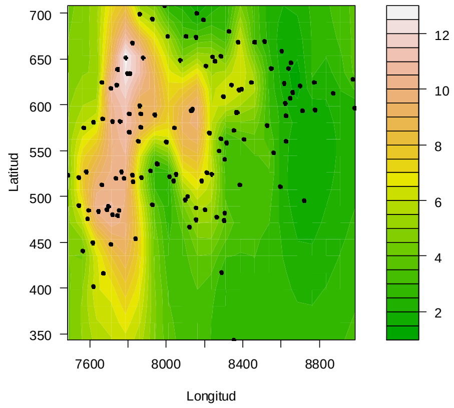
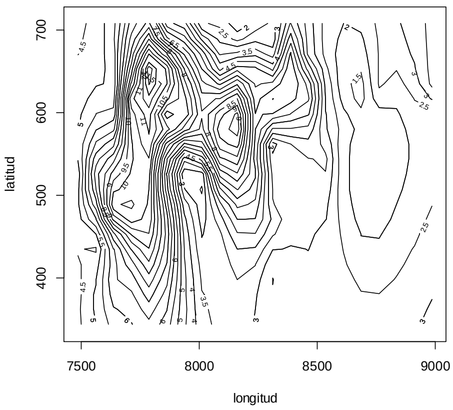

Latitud
Longitud

KOLSS 15 500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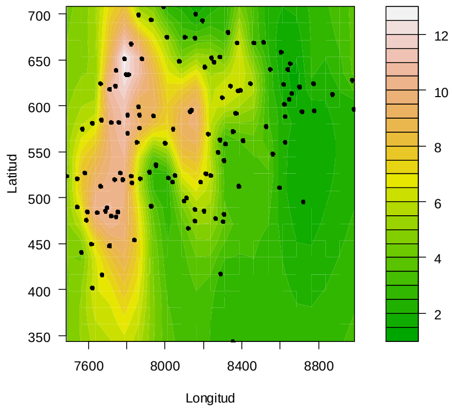
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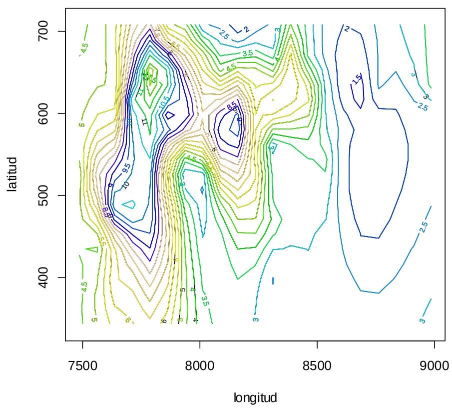
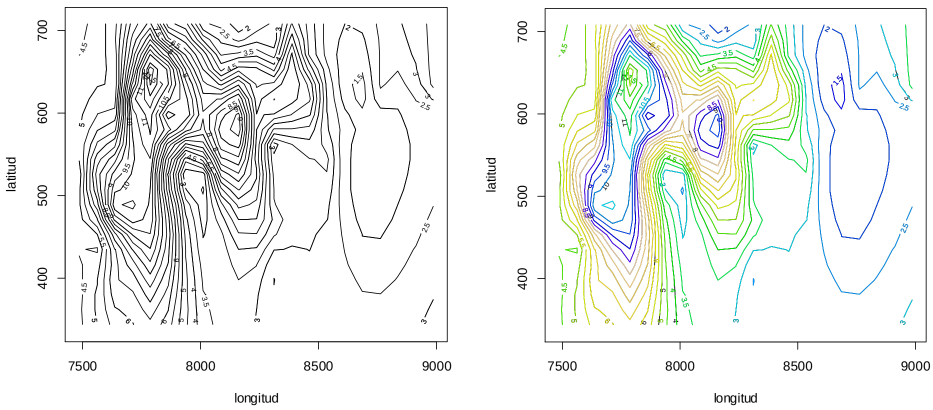
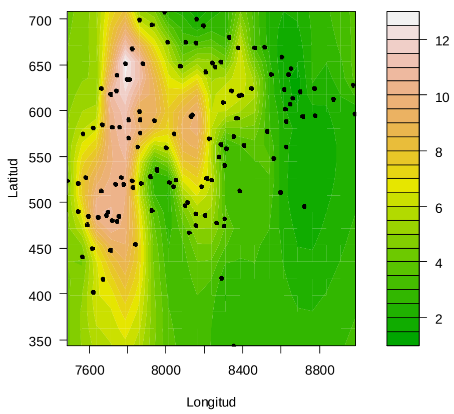
Longitud



KWLSS 25 500





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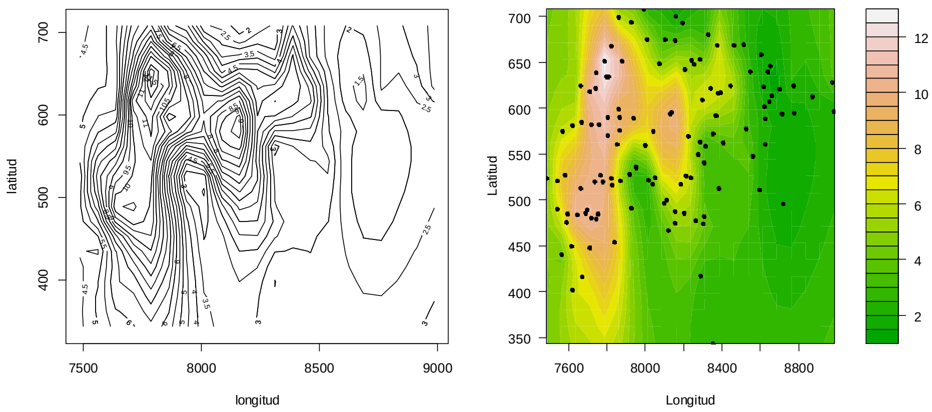

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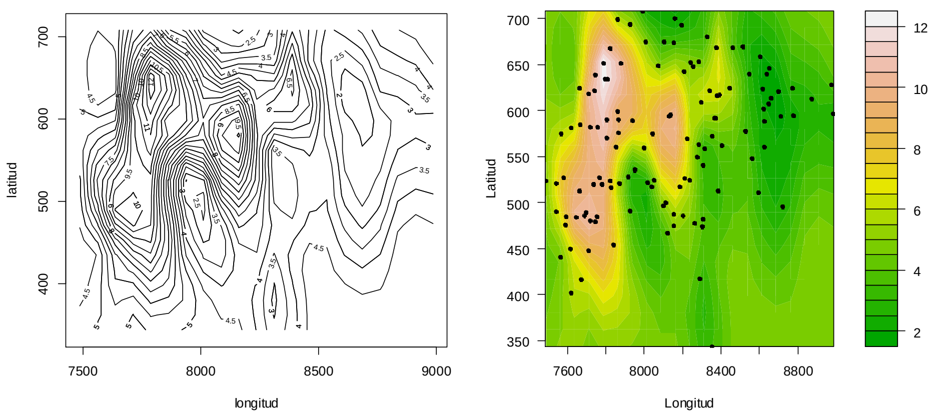

KWLSS 35 500

Pada kes endapan nikel laterit, kesan nugget atau berlakunya variabiliti data yang tinggi pada jarak dekat ini diindikasi oleh Heriawan, dkk (2009) sebagai berkaitan dengan pengayaan mineralisasi pada zon rekahan (*fracture*).