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Kriging by partition: case of Ciurug Quartz Gold Vein

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Abstract. This paper presents the calculation of reserves gold distribution using ordinary kriging with regional partition on the clustering of distribution zone base. The calculations of resources or reserves in mining industry depend, among other things on sample point accuracy prediction. As a part of geostatistics, kriging is a good and unbiased predictor which is based on robust semivariogram, and fitting here using ordinary least squares (OLS). However, this need foresight in determining method or pattern of sorts which is more appropriate to be used. Based on the partitioning method, it found that gold grade average is higher than the original base as shown in each of the mean value.

Keywords: Semivariogram, OLS fitting, partitioning, kriging.

1. Introduction

Kriging is a geostatistical method in which used to predict the unsampled points referred to known samples as information base. Kriging method, especially in mining industry, used for reserves calculations [1, 2, 3]. The main foundations of kriging are nugget, sill and range, i.e., semivariogram parameters obtained by empirical semivariogram fitting on semivariogram theory. Two empirical semivariograms which commonly has been used are classical model of [4] and robust model of [5]. The last model is a refinement of the classical model often prone to atypical case of data contamination, in which statistically can be seen in skewness value. As a spatial prediction base, semivariogram should have conducted an examination of the raw data against the possibility of a trend and outliers [6]. In this study, kriging parameters were obtained using the [5] model which identified as a robust variogram.

The fitting processes in a narrow sense known as the trend line built by semivariogram discrete curves into the theoretical continuous function appropriately. In mining industry, especially as a basic of resources or reserves prediction, the theoretical model tendency as the transition model that has a parameter sill (i.e., spherical, experimental, gaussian, etc.) used. This model closely related to obtain of range, i.e. a parameter which is indicates of maximum distance between the spatial influences of point. Sill mean as a semivariogram indicates the maximum spatial variance effects between the points, and afterwards, the influences between the points are no longer.



2. Methods and materials

Metal ore, especially gold grade precipitated in a vein system are not uniformly distribute, even in each region was often erratical [7]. The distribution of data can be determined through statistical processing, one of which is skewness as a value indicates the presence of outliers data [6].

2.1 Geostatistics

Geostatistics is the science of statistics application, based on the concept of probability of a natural phenomenon, which is the formal application of the random function. This terminology stems from the study which was done to ore deposits (gold) characterized by the spatial distribution of a regionalized variable values [8,9].

2.2 Empirical semivariogram

If $Z(\mathbf{s}_i)$ is the value of a variable (say, the gold grade) located in a location \mathbf{s}_i , $Z(\mathbf{s}_j)$ is the value of variable (also grade gold) located at \mathbf{s}_j or, which is distance \mathbf{h} from $Z(\mathbf{s}_i)$, whereas $N(\mathbf{h})$ is the number pairs of sample points on the lag \mathbf{h} , then the empirical semivariogram estimator (known as classical semivariogram), is a function which is formulated [10] as

$$\hat{\gamma}(\mathbf{h}) = \frac{1}{2|N(\mathbf{h})|} \sum_{i=1}^{N(\mathbf{h})} [Z(\mathbf{s}_i) - Z(\mathbf{s}_j)]^2 \quad (1)$$

Function as Eq. (1), assessed by Cressie and Hawkins (1980) [5], as a highly vulnerable to the emergence of atypical observations, so that through the Box & Cox transformation [11] and assumes a principle to normality, then refine it by presenting a more robustsemivariogram estimator (known as robust estimator) as

$$\bar{\gamma}(\mathbf{h}) = \left(\frac{1}{2|N(\mathbf{h})|} \sum_{i=1}^{N(\mathbf{h})} [Z(\mathbf{s}_i) - Z(\mathbf{s}_j)]^{1/2} \right)^4 / \left(0.457 + \frac{0.494}{|N(\mathbf{h})|} \right) \quad (2)$$

2.3 Semivariogram fitting

Semivariogram estimator as Eq. (1) and Eq. (2) is an empirical (discrete), so, to obtain further information required of fitting with theoretical seivariogram function. Fitting method in this paper is using ordinary least squares (OLS) with the formula [12]

$$\min \sum_{i=1}^k [\hat{\gamma}_z(\mathbf{h}_i) - \gamma_z(\mathbf{h}_i; \theta)]^2 \quad (3)$$

The information obtained from fitting calculation, especially, is sill and range (of influence). Sill which is geologically understood as igneous intrusions concordant to bedding intruded rocks [13] geostatistically understood as a condition in which the semivariogram has reached a stationary condition. While the range practically, is the maximum distance semivariogram to reach the sill, or written as

$$\lim_{|\mathbf{h}| \rightarrow \infty} \gamma(|\mathbf{h}|) = \gamma_\infty < \infty \quad (4)$$

As widely used in mining, this paper used spherical semivariogram models

$$\gamma(\mathbf{h}) = \begin{cases} c \left\{ \frac{3\mathbf{h}}{2a} - \frac{1}{2} \left(\frac{\mathbf{h}}{a} \right)^3 \right\}, & \mathbf{h} \leq a \\ c, & \mathbf{h} > a. \end{cases} \quad (5)$$

\mathbf{h} is the lag distance, a is the range (of influence) while c is the sill. Execution data in this study is using the R package[14].

2.4 Ordinary kriging

Ordinary kriging (OK) is a linear method with a weighted-average which serves as a predictor of the value at an unknown point, taking into the account type of data on the other points which has been known. Suppose $Z(s_1), Z(s_2), \dots, Z(s_n)$ is a collection of grade goldpoints which is at the spatial points s_1, s_2, \dots, s_n , and s_0 is the prediction point, then the results of grade predicted of $\hat{Z}(s_0)$ is defined as [10]

$$\hat{Z}(s_0) = \sum_{i=1}^n w_i Z(s_i). \tag{6}$$

Unbiasedness term is true if $\sum_{i=1}^n w_i = 1$.

Kriging method is used to minimize the mean squared error prediction

$$\min \sigma_e^2 = E[Z(s_0) - \hat{Z}(s_0)]^2 = \min \left[Z(s_0) - \sum_{i=1}^n w_i Z(s_i) \right]^2 \tag{7}$$

In the case where Z is an intrinsically stationary process, the variance is defined as

$$\sigma_e^2 = E[Z(s_0) - \sum_{i=1}^n w_i Z(s_i)]^2 = 2 \sum_{i=1}^n w_i \gamma(s_0 - s_i) - \sum_{i=1}^n \sum_{j=1}^n w_i w_j \gamma(s_i - s_j) \tag{8}$$

Provided that,

$$\sum_{i=1}^n w_i = 1. \tag{9}$$

Minimization is done for (w_1, w_2, \dots, w_n) satisfying of the boundary condition $\sum_{i=1}^n w_i = 1$. To minimize is,

$$\min 2 \sum_{i=1}^n w_i \gamma(s_0 - s_i) - \sum_{i=1}^n \sum_{j=1}^n w_i w_j \gamma(s_i - s_j) - 2\lambda \left(\sum_{i=1}^n w_i - 1 \right), \tag{10}$$

λ , is Lagrange multiplier. Further more,

$$\frac{\partial \sigma_e^2}{\partial w_i} = 0 = 2\gamma(s_0 - s_i) - \sum_{j=1}^n w_j \gamma(s_i - s_j) - 2\lambda$$

Thus obtained

$$\sum_{j=1}^n w_j \gamma(s_i - s_j) + 2\lambda = 2\gamma(s_0 - s_i), \quad \forall i = 1, \dots, n. \tag{11}$$

Whereas,

$$\frac{\partial \sigma_e^2}{\partial \lambda} = 0 = 2 \left(\sum_{i=1}^n w_i - 1 \right) \rightarrow \sum_{i=1}^n w_i = 1.$$

If the matrix $\mathbf{W} = (w_1, w_2, \dots, w_n, \lambda)$ and $\boldsymbol{\gamma} = (\gamma(s_0 - s_1), \gamma(s_0 - s_2), \dots, \gamma(s_0 - s_n))'$,

$$\boldsymbol{\Gamma} = \begin{cases} \gamma(s_i - s_j), & i = 1, 2, \dots, n, \quad j = 1, 2, \dots, n, \\ 1, & i = n + 1, \quad j = 1, \dots, n, \\ 1, & j = n + 1, \quad i = 1, \dots, n, \\ 0, & i = n + 1, \quad j = n + 1. \end{cases} \tag{12}$$

concisely written as $\boldsymbol{\Gamma} \mathbf{W} = \boldsymbol{\gamma}$. Thus the weight of w_1, w_2, \dots, w_n and λ can be calculated by

$$\mathbf{W} = \mathbf{\Gamma}^{-1}\boldsymbol{\gamma}. \tag{13}$$

Variance predictor, at last presented as

$$\sigma_e^2 = \sum_{i=1}^n w_i \gamma(\mathbf{s}_i - \mathbf{s}_0) + \lambda \tag{14}$$

2.5 Partitioning

A location can be divided into several sub-locations, so semivariogram at prime location, of course, will vary with the semivariogram from smaller sub-locations. OK models typically used for large location without having to pay attention to sublokasi. OK models in this study extended to the use of sub-locations.

2.6 Matrix of two partitions

Suppose, that the main matrix has the form of [15]

$$\mathbf{A} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1n} \\ Z_{21} & Z_{22} & \dots & Z_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{m1} & Z_{m2} & \dots & Z_{mn} \end{bmatrix}. \tag{15}$$

The matrix is then partitioned into two parts, namely, M as a matrix to partition-1 and N is the matrix Partition-2, where

$$\mathbf{M} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1k_1}' \\ Z_{21} & Z_{22} & \dots & Z_{2k_1}' \\ \vdots & \vdots & \ddots & \vdots \\ Z_{k_1 1} & Z_{k_1 2} & \dots & Z_{k_1 k_1}' \end{bmatrix} \text{ and } \mathbf{N} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1k_2}' \\ Z_{21} & Z_{22} & \dots & Z_{2k_2}' \\ \vdots & \vdots & \ddots & \vdots \\ Z_{k_2 1} & Z_{k_2 2} & \dots & Z_{k_2 k_2}' \end{bmatrix}. \tag{16}$$

Each sub-location has data observations as Z_{11} , with range of a , and the sill, c .

2.7 OK with matrix of two partitions

Suppose, that the data will be used to predict the value at the point \mathbf{s}_0 are on location-1 and location-2, then the predictor involves two partition matrix, i.e., M and N matrixes. Thus, there will be only two semivariogram where the estimator is optimal, if

$$\hat{Z}(\mathbf{s}_0) = \sum_{i=1}^{k_1} a_i Z_i + \sum_{s=1}^{k_2} b_s Z_s. \tag{17}$$

$\hat{Z}(\mathbf{s}_0)$ is the predicted value at the point \mathbf{s}_0 , a_i is the weight assigned to the sample point Z_i ($i=1, \dots, k_1$), and b_s is the weight assigned to the sample point Z_s ($s=1, \dots, k_2$).

If the actual value of \mathbf{s}_0 , is $Z(\mathbf{s}_0)$, the error variance is defined as

$$\begin{aligned} \sigma_e^2 &= E[Z(\mathbf{s}_0) - \hat{Z}(\mathbf{s}_0)]^2 = E[Z(\mathbf{s}_0) - \sum_{i=1}^{k_1} a_i Z_i - \sum_{s=1}^{k_2} b_s Z_s]^2 \\ &= -2 \sum_{i=1}^{k_1} a_i \gamma(\mathbf{s}_0 - \mathbf{s}_i) - 2 \sum_{s=1}^{k_2} b_s \gamma(\mathbf{s}_0 - \mathbf{s}_s) + \sum_{i=1}^{k_1} \sum_{j=1}^{k_1'} a_i a_j \gamma(\mathbf{s}_i - \mathbf{s}_j) + \sum_{s=1}^{k_2} \sum_{j=1}^{k_2'} b_s b_j \gamma(\mathbf{s}_s - \mathbf{s}_0) \end{aligned}$$

In order for the OK method generates a prediction with minimum error variance and un-biased, it is necessary to add the Lagrange multiplier (λ) where the number of weights in all locations equals to one, so that

$$\sigma_e^2 = -2 \sum_{i=1}^{k_1} a_i \gamma(\mathbf{s}_0 - \mathbf{s}_i) - 2 \sum_{s=1}^{k_2} b_s \gamma(\mathbf{s}_0 - \mathbf{s}_s) + \sum_{i=1}^{k_1} \sum_{j=1}^{k'_1} a_i a_j \gamma(\mathbf{s}_i - \mathbf{s}_j) + \sum_{s=1}^{k_2} \sum_{j=1}^{k'_2} b_s b_j \gamma(\mathbf{s}_s - \mathbf{s}_0) - 2\lambda \left(\sum_{i=1}^{k_1} a_i + \sum_{s=1}^{k_2} b_s - 1 \right). \tag{18}$$

The minimum error variance obtained by the first partially derivated (with respect to a, b and λ) in which, each equal to zero.

$$\frac{\partial \sigma_e^2}{\partial a_i} = 0 \text{ thus obtained } \sum_{j=1}^{k'_1} a_j \gamma(\mathbf{s}_i - \mathbf{s}_j) + 2\lambda = 2\gamma(\mathbf{s}_0 - \mathbf{s}_i), \quad \forall i = 1, \dots, k_1.$$

$$\frac{\partial \sigma_e^2}{\partial b_s} = 0 \text{ thus obtained } \sum_{j=1}^{k'_2} b_j \gamma(\mathbf{s}_s - \mathbf{s}_j) + 2\lambda = 2\gamma(\mathbf{s}_0 - \mathbf{s}_s), \quad \forall s = 1, \dots, k_2.$$

$$\frac{\partial \sigma_e^2}{\partial \lambda} = 0 \text{ thus obtained } \sum_{i=1}^{k_1} a_i + \sum_{s=1}^{k_2} b_s = 1.$$

In a matrix, the OK system is written as [16]

$$\begin{bmatrix} \gamma(\mathbf{s}_1 - \mathbf{s}_1) & \dots & \gamma(\mathbf{s}_1 - \mathbf{s}_{k_1}) & 0 & \dots & 0 & 1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \gamma(\mathbf{s}_{k_1} - \mathbf{s}_1) & \dots & \gamma(\mathbf{s}_{k_1} - \mathbf{s}_{k_1}) & 0 & \dots & 0 & 1 \\ 0 & \dots & 0 & \gamma(\mathbf{s}_1 - \mathbf{s}_1) & \dots & \gamma(\mathbf{s}_1 - \mathbf{s}_{k_2}) & 1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & 0 & \gamma(\mathbf{s}_{k_2} - \mathbf{s}_1) & \dots & \gamma(\mathbf{s}_{k_2} - \mathbf{s}_{k_2}) & 1 \\ 1 & \dots & 1 & 1 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ \vdots \\ a_{k_1} \\ b_1 \\ \vdots \\ b_{k_2} \\ \lambda \end{bmatrix} = \begin{bmatrix} \gamma(\mathbf{s}_0 - \mathbf{s}_1) \\ \vdots \\ \gamma(\mathbf{s}_0 - \mathbf{s}_{k_1}) \\ \gamma(\mathbf{s}_0 - \mathbf{s}_1) \\ \vdots \\ \gamma(\mathbf{s}_0 - \mathbf{s}_{k_2}) \\ 1 \end{bmatrix} \tag{19}$$

With a similar path can be searched,

$$\mathbf{W} = \Gamma^{-1} \boldsymbol{\gamma}. \tag{20}$$

2.8 Data

Gold distribution which has sedimented in the vein quartz (in g/t Au) obtained by the assaying of 138 drilling points, in Pongkor Ciurug region, i.e., the area of PT. Aneka Tambang UBPE Pongkor, West Java, Indonesia [17].

Based on statistical calculations, the distribution of data can be grouped into two parts. First is the high-spread distribution grade in the western area. Whereas the second is a low grade distribution area values, spread to the east. Partitioning divisibility of the two regions, are taken into two considerations. Partition 1 is the rich zone in the western region which consists of 78 data, while Partition 2 which consists of 50 data resides in the eastern region, commonly is a poor zone.

3. Results And Discussions

The initial step in spatial analysis is to examine the raw data for the presence of trends and outliers [18, 19,20]. Based on skewness of 2.5, then 10 outlier data was eliminated (> 15 g/t Au). Another consideration is also associated with the value of CV = 0.89 (already, close to one), the value of kurtosis is high enough (12.41), and the difference between the mean and maximum values which are also large (i.e. 42.250-7.055=35.195 g/t Au).

Statistic calculations on the data as much as 128 (where outliers have been eliminated), provide information as Table 1. Original is a spatial data sets that consist of 128 sample points, while the

Partition-1 and Partition-2, is the result of data partition, where the amount of data, respectively 78 and 50.

Table 1. Statistic parameters of gold grade (in g/t Au)

Location	N	Min	1 st Qu	Median	Mean	3 rd Qu	Max	Skewness
Original	128	0.450	2.650	5.275	5.823	8.362	14.800	0.602
Partition 1	78	0.450	4.320	6.975	7.349	10.630	14.800	0.193
Partition 2	50	0.500	1.650	2.600	3.442	5.275	5.275	0.649

It appears that, Original and Partition-1 as in third column of row 2 and row 3 of **Figure 1**, has a same minimum data value, which is 0.45 g/t Au. Maximum data is same, namely 14.80 g/t Au. While maximum of Partition-2 data is 5.275 g/t Au, although, the minimum data is 0.5 g/t, i.e., slightly larger than minimum data of Partition-1. This suggests, that low grade, relatively more in the east zone. In contrast, the Partiton-2 more in the west zone.

3.1 Semivariogramandfitting

Semivariogram is the main tool of the regionalized variable theory, which is quantifies the size and intensity of spatial variation as well as provide a basis for the optimum interpolation through kriging method [18]. ased on many simulations, semivariogram in this study than assumed as omnidirectional.

Fitting semivariogram is the second important step on the spatial prediction, because it plays a role in determining of the kriging weights. Original data ($n=128$) semivariogram calculation (as Eq. 2) then fitted using the OLS (Eq. 3), which is the lag distance of 50 meters, and the number of 30 pairs of lag earned 8128 points. On the assumption of zero nugget, obtained sill is 24.684 (g/t)² Au, and the range (maximum spatial effect between the point) is 988.593 m (**Figure 1**).

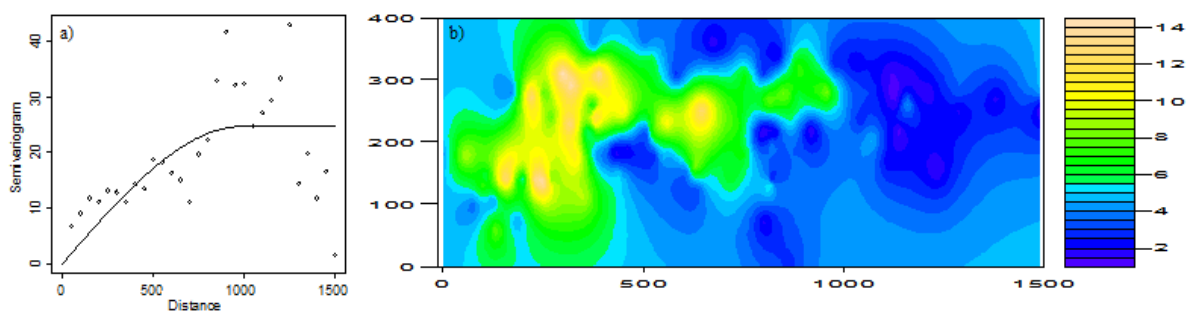


Figure 1. (a) Spherical fitting of Original ($n=128$, sill=24.68, range=988.593) (b) OK Predicted based on Spherical Model of Original based on a)

Figure 2 illustrates the semivariogram trip of Partition-1 (**Figure 2a**) and Partition-2 (**Figure 2b**). On Partition-1, semivariogram reaches the sill which is 17.293 (g/t)² Au, at around of lag 6 and lag 7, with a maximum distance of influence (or range) is 163.348 m. Starting from lag 10 to lag 16, fluctuations occur up of the sill, except in lag 13 (i.e., 16.420).At a distance over 400 m, there is an interesting phenomenon where the values of semivariogram continued decline to the lowest point, even far below the semivariogram value at the first lag. Very likely, this occurs because of changes in the structure, or the variations in the form of veins located in the western zone.

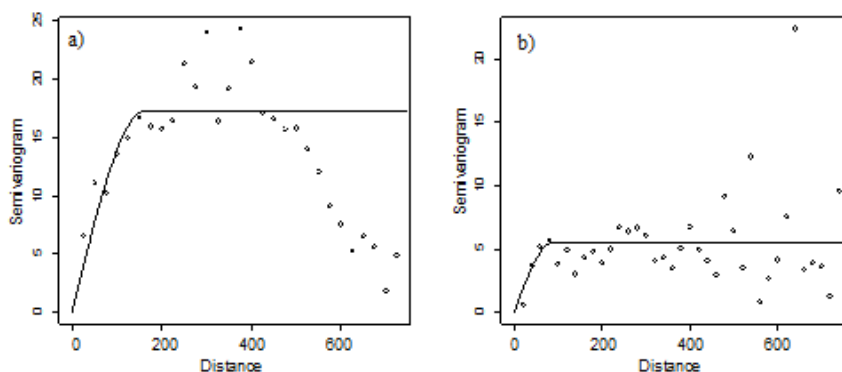


Figure 2. Spherical fitting of (a) Partition-1 (sill=17.293, range=163.348) and (b) Partition-2 (sill=5.528, range=88.486)

Figure 2b, represents the semivariogram of Partition-2, which consists of 50 data samples. OLS fitting with spherical models, produced the effect of spatial point in a range of 88.486 m, where the mean sill is 5.528 (g/t)² Au which lasted until the distance nearly 500 m.

As shown in (mainly) **Figure 2a**, where a very sharp decline occurred on semivariogram value after a lag of 400, it is likely due to the indications of geological phenomena, say the structure changes or other phenomena [20].

3.2 Kriging

Kriging prediction (OK) of three sets of data, (i.e. Original, Partition 1 and Partition 2) based on the results of spherical model fitting parameter, where the size of the grid respectively 5×5. Regions of Original has an area of 547.500 m², whereas partion Partition-1 and-2, respectively 228.750 m² and 252.750 m².

Grade distribution (as shown in **Figure 3**) predicted by ordinary kriging with grid of 5×5. **Figure 2** shows the statistical data and kriging prediction of Original (with $n=128$ data), Partiton-1 ($n=78$) and Partition-2 ($n=50$).

Data on Original and Partition-1 predicted (by ordinary kriging) higher than each of the true data. If considered carefully (especially on mean), it is seen that the true of Original data (i.e. 5.823 g/t Au) is predicted to be 5.846 g/t Au (up 0.4%). As for Partition-1 which was originally 7.349 g/t Au, predicted with 7.407 g/t Au (or rise of 0.8%). Kriging variance for the Original is 1.26 g/t Au and Partition-1 is 5.09 g/t Au.

Table 2. Statistic of Non-predicted and OK Predicted (in g/t Au)

Location	N	Source	Min	1 st Qu	Median	Mean	3 rd Qu	Max
Original	128	True data	0.450	2.650	5.275	5.823	8.362	14.800
		Predicted	0.939	2.956	5.681	5.846	7.901	14.110
		KrigeVar	0.001	0.704	1.207	1.259	1.684	6.078
Partition-1	78	True data	0.450	4.320	6.975	7.349	10.630	14.800
		Predicted	0.774	5.517	7.334	7.407	9.723	14.110
		KrigeVar	0.003	2.943	5.163	5.095	7.136	12.190
Partition-2	50	True data	0.500	1.650	2.600	3.442	5.275	5.275
		Predicted	0.486	2.484	3.312	3.441	4.270	6.770
		KrigeVar	0.045	1.888	3.587	3.193	4.775	5.770

Unlike the above of two locations, Partition-2 estimated almost same, namely of true data, i.e. 3.442 g/t Au predicted to be 3.441 g/t Au. However, the mean value of the kriging variance of Partition-2 is

quite high (i.e. 3.193), or close to the data mean 3.4 g/t Au. The Average of mean value prediction of Partition-1 and Partition-2 is 5.424 g/t Au. This value is slightly lower than the Original Data (5.486 g/t Au).

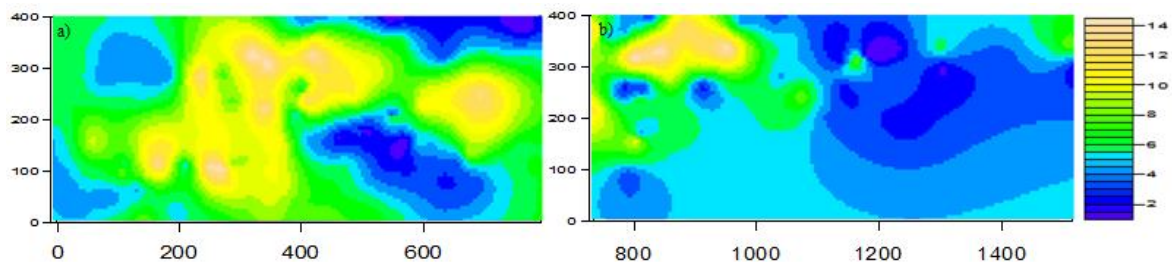


Figure 3. OK Predicted based on Spherical Model of (a) Partition-1 ($n=78$, $c=17.293$, $a=163.348$) and (b) Partition-2 ($n=50$, $c=5.528$, $a=88.486$). As in the description, yellow and beige color show high gold grade.

4. Conclusion

In general, ordinary kriging (based on robust semivariogram) gives a higher prediction than the true data. But, kriging variance of Original looks better than the kriging variance of partitioning results (Partition-1 and Partition-2). Although, the average value of prediction results of partitioning is slightly lower than the predicted value of Original data. Partitioning provides an advantage, especially, on the distribution of data which form a self-contained zone (or, rich-zone and-poor zone) and it is useful to determine priorities in the plan of mining exploitation.

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