# Mineral Resource Estimation Using Weighted Jackknife Kriging

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**Abstract**. The handling of the high grade as an outlier on precious metals deposits is very important in geostatistical ore resources estimation. Ordinary kriging (OK) that is very popular used to estimate the grade of ore resources is often inaccurate in handling of the outlier. This paper applied the method of weighted jackknife kriging (WJK) as a practical method to deal with the problem of high grade outlier. The simplicity concepts in mathematics and robustness are the attraction of the method in geostatistical ore resources estimation.

## INTRODUCTION

In mineral deposits with highly skewed grade distributions, such as precious metal deposits, proper handling of the high grade outlier data is very crucial to the geostatistical ore reserve estimation. By outlier high grade datum, it is understood a datum much higher than the median or mean of other data, and actual reliable value corresponding to a known mineralization, not an assay error or a misplaced decimal point. Such outlier data may represent only a minute percentage of the total data set but can contribute a large percentage of the total in situ quantity of metal [1].

In precious metal deposits with highly skewed data, high grade often are mixed with waste, and in the absence of a clear geologic or structural control, the OK or conventional approach is either to smear these high grades over a wide interval, to cap them, or to discard them altogether as outliers [2]. There are some recent kriging techniques, such as indicator or probability kriging, which are available to deal with the problems associated with the outlier data. However, these methods are either too complex or too exhaustive for mine engineers and geologists who often do not have the expertise or the time to apply them to their problem [3]. In traditional estimation methods where grades at unsampled locations are estimated using surrounding samples, each sample is used only once for every location being estimated. Jackknifing the estimate is a simple way to maximize sample utilization.

The weighted jackknife represents another estimator of a good, simple idea. This paper examines the results of using the WJK and OK of its application to a gold porphyry deposit.

## OBJECTIVE

The objective of the research is: (a) to introduce a tool that can be useful to apply in mineral resource estimation of some deposits with highly skewed distribution. The estimator retains the simplicity of the original OK estimator, and is more robust for a wide range of data distribution, (b) to evaluate the results of using the WJK and OK of its application to a gold porphyry deposit. Jackknifing the OK estimate can yield a potentially more accurate result by reducing local bias, while at the same time retaining OK's global unbiasedness property.

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## METHODS AND MATERIAL

### Basic Concept

Weighted jackknife is based on resampling scheme. Originally developed to reduce bias, further developments allow it to quantity the uncertainty associated with the estimation [4-7]. Jackknifing the grade estimate allows valuable information to be used more than once for each location. Since estimated grade is a function of available samples, we can improve our estimation not only by using information from a particular sample, but also by using information developed when the sample is absent.

The basic idea of the jackknife is very simple. First, an estimate is obtained using all data. Then, a sample is removed from the data and a new estimate is obtained using the remaining samples. This process is repeated until every sample has been, in turn, removed. These new estimates are then combined with the original estimate, weighted in such a way to produce cancellation of their biases. More importantly, jackknifing reduces bias at the local level. Hence, jackknifing the OK estimate is a very attractive proposition.

The weight eventually assigned to each sample by the jackknife is a linear combination of the original kriging weight when all n samples are used and the kriging weight when all except the it sample are used, i = 1, …, n. The result is redistribution of weights whereby the closest samples are assigned most of the weight, when down weighting the influence of samples that are farther away, which often are assigned small negative weights. This gives the jackknife OK estimate a less smooth appearance, and a tendency to follow local grade distribution better than does the original OK estimates. Not only does the estimator retain the global unbiasedness property of OK, it is also more robust than OK because it works for a wider range of data distribution. At the same time, it still retains all of OK's simplicity.

*Weighted jackknife kriging formulation*

Let $Z(s_1)$, $Z(s_2)$, …, $Z(s_n)$ be independent, identically distributed value of a sample of size n [8]. Denote $\hat{Z}(s_0)$ as an estimator of the parameter Z, the result of making a complex calculation using all n samples. Here $\hat{Z}(s_0)$ would be the OK estimate of the true grade Z. Let $\hat{Z}(s_{0,-i})$ be the corresponding estimate when the sample $\hat{Z}(s_i)$ is omitted. The $i^{th}$ weighted pseudo value is defined as:

$$\hat{Z}^{w}_{p,-i} = \hat{Z}_0 + n(1-w_i)(\hat{Z}_0 - \hat{Z}_{0,-i})_{i=1,2..n} \tag{1}$$

The details of the formulations are given in [9].

The weighted jackknife estimator is the mean of the weighted pseudo value:

$$\hat{Z}^{w}_{jack} = \frac{1}{n}\sum_{i=1}^{n} Z^{w}_{p,-i} \tag{2}$$

The weighted jackknife standard deviation is:

$$\sigma_{jack} = \left[\frac{1}{n(n-1)}\left\{\sum_{i=1}^{n} {Z^{w}_{p,-i}}^{2} - \frac{1}{n}\left(\sum_{i=1}^{n} Z^{w}_{p,-i}\right)^{2}\right\}\right]^{\frac{1}{2}} \tag{3}$$

Equation (3) can be used as a means to construct a confidence interval for the weighted jackknife estimate. Discusses the application of the jackknife standard deviation in estimating grade of a volume much larger than a block, for example calculating a confidence limit for ore reserves are given in [3,9].

## RESULT

### Example

Let us take a hypothetical case shown in Fig. 1(a). There are 11 data points in this figure, including one outlier datum of 1.019 oz/t. The mathematical average of all data is 0.103 oz/t. This average drops to 0.027 without the outlier datum. Thus, one outlier datum, which in this case is 9% (1/11) of all data, contributes 76% to the total metal content of the data points. In other words, there is no excitement without the outlier datum.

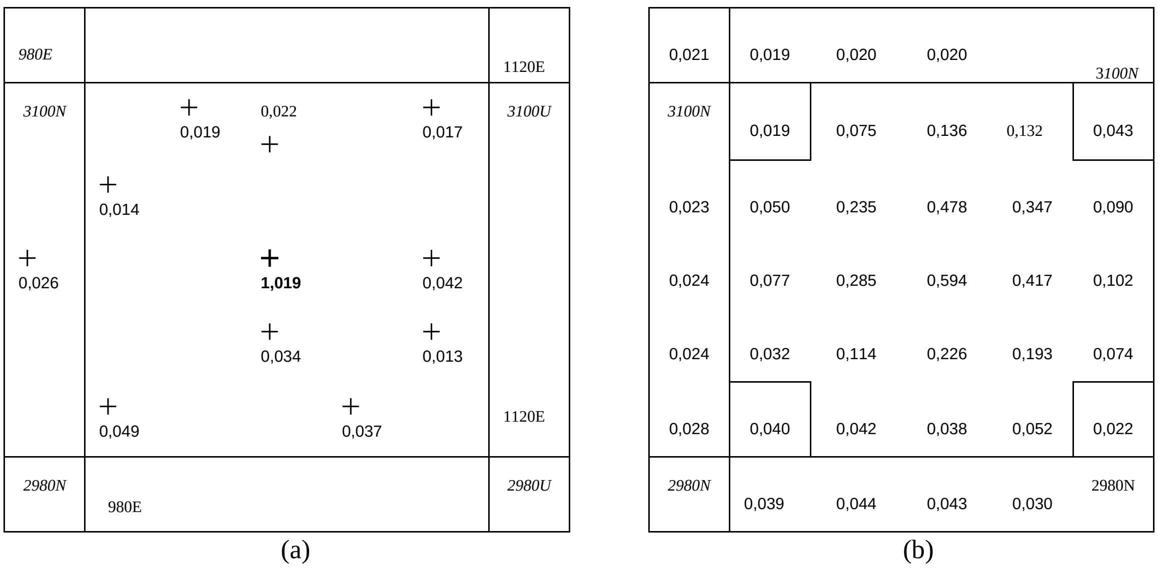



**FIGURE 1**. Gold bench composite data [1] showing a high grade outlier (a), and the OK estimate (b), indicating the high grade smearing to the outlying blocks

Now let us draw a circle of 50 m radius from the center of the block that contains the outlier datum, and display all the blocks whose contents fall within the circle. These are the block that might be affected the most from the outlier datum if it is included in the interpolation of the block values.

*OK application*

Let us first interpolate these block using the OK method. The variogram parameter used in this interpolation and the resulting block value are: C0 = 0.0005, C1 = 0.0045, range = 100 m. A cutoff grade of 0.030 oz/t is outlined in these estimates. The average value of the block is 0.180 oz/t, which is about 82% higher than the average of the data used to interpolate these block. Thus, the outlier datum has significantly influenced the value of the blocks within its specified range, causing the over-valuation of some or all of the blocks. Unfortunately the OK cannot help prevent the smothering of the outlier data into the surroundings, since its weighting scheme is independent of the data values. The estimates in Fig. 1(b) to 2 are obtained using uniform search distance of 50 m, minimum of 3 composites and maximum of 15 composites.

*WJK application*

Fig. 2 shows the WJK estimate of the same block. Jackknifing succeeded in limiting the smearing of high grade from the one high grade composite. Not only is the high grade smearing reduced, but also this estimator follows the data much more closely. Beyond the 50 m search radius from the high grade outlier, i.e. in the peripheral blocks, the grades interpolated by the two methods are very similar.

A global comparison between the data and the estimates is instructive. The arithmetic average of all 11 composites is 0.103 oz/t, with a CV (coefficient of variation) of 2.69. The mean block grade and CV of the 21 center blocks within 50 m radius from the high grade outlier composite are as follow: using OK estimator 0.180 oz/t and 0.89; using WJK estimator 0.103 oz/t and 1.81. Clearly, OK predicts much more gold that the data would indicate, while the WJK estimate is closer to the mean of the data.

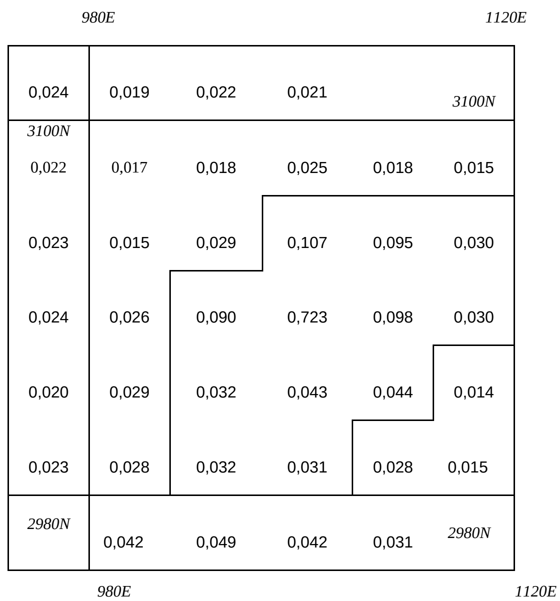



**FIGURE 2**. The WJK estimate for data configuration shown in Fig. 1(a). WJK reduces the high grade smearing and gives an estimate that follows data trend better

Further analysis by applying a cutoff to these blocks located within 50 m from the outliers more illuminating. A cutoff grade of 0.030 oz/t applied to the OK estimate will send all 21 blocks to the mill. The same cutoff applied to the WJK estimate results in only 12 blocks above cutoff. This example illustrates two important aspects of grade estimation: a reliable estimate of block grades from exploration data to form a sound basis for economic analysis and mine planning, and an equally reliable estimate from blast hole data to avoid misclassification of ore and waste.

## DISCUSSION

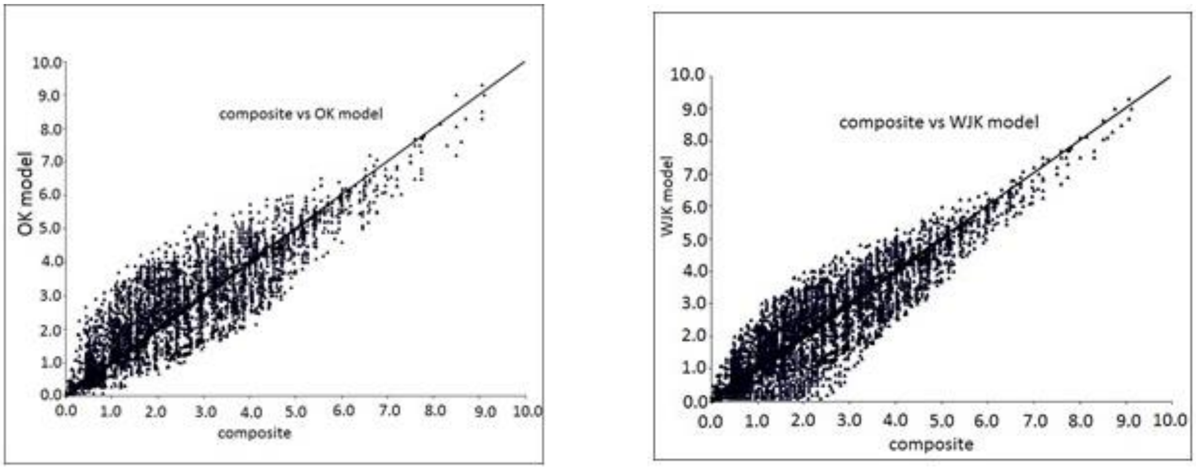
The following discussion is the comparison of application methods of OK and WJK on gold porphyry. The deposit studied is approximately 1000 m x 1000 m in plan. The genesis of the gold porphyry is interpreted as a series of multiple intrusions that are superimposed with multiple pulses of mineralized fluids. The geometry of the intrusion is generally concentric in plan with extensive vertical continuity. The timing of the mineralizing pulses have been interpreted to occur between the intrusive events so that some mineralization is post intrusion, some simultaneous to intrusion, and some remobilized or removed by later stage intrusive. Mineralization within the deposit is comprised of a series of spatially superimposed mineralizing events that can occur singly or in combination. The highest grade zone of the deposit is a quartz-sulfide stock work zone, which is horseshoe shaped in plan and cylindrical in section.

A total of 50,718 samples transformed data from 407 drill holes are used in this study contain the information of Au grades, specific gravity and rock types. The rock types are grouped into 3 major rock type based on the mineralization/grade distribution inside. The grouped are Hornblende Diorite, and layer of Volcanic Pyroclastic, and group of Sediment.

Most drill holes in the area are inclined, and drilled to cross the mineralization. There are also horizontal hole from the old underground working. The data are then composited in geology break down, 15 m lengths as the bench height of open pit design. A total of 11,443 samples were left after composited. Below detection limit of the data are converted into positive value of half detection limit.

The 3D block model built for the deposit consists of 20 m x 20m x 15 m size blocks, where 20 m is the side of a block in the E-W and N-S directions. The bench height is 15 m. Each block is assigned a rock code indicating whether the block is inside ore or outside the mineralization. The blocks outside the mineralization are not interpolated.

Fig. 3 shows the x-y plots of the gold composite vs. estimates grades at the same location, together with the idealized 45 degree line. The linear regression statistics are given in Table 1. These statistics indicate comparable performance between the two estimators. However, visual observation of the x-y plots is more instructive. The WJK estimates follow the 45 degree line (bisector) slightly better than does the OK estimate.




(a) (b)

**FIGURE 3**. Scatter plot of gold composite vs. estimated grade, showing the first bisector (45 degree) line. The gold composite is plotted against the OK estimate (a), the WJK estimate (b)

This is also indicated by their lower intercept and higher slope compared to OK (Table 1). The example shows that even in a symmetrically distributed data set favorable for OK, the WJK estimator performs well or slightly better than the OK estimation.

**TABLE 1.** Linear regression statistics: X-Y plot of gold composite vs. estimated grades at all the same location

| Parameter | Ordinary kriging | Weighted jackknife kriging |
|---|---|---|
| Intercept | 0.209 | 0.147 |
| Slope | 0.791 | 0.848 |
| SEE | 0.612 | 0.496 |
| R | 0.670 | 0.800 |

## CONCLUSION

WJK is a technique in geostatistical concepts that can be applied to estimate grade of mineral resources which have high grade outlier. This study provides conclusions that jackknifing OK estimates can produce a more accurate grade estimate, and at the same time retaining global unbiasedness. WJK method remain uses the concept of OK simplicity, but produces more accurate estimates when compared with the OK method.

Further research is necessary in various types of data for the evaluation of the application and possible limitations. WJK method adds alternative options in the valuation grade of mineral resources and solve the problem of data with highly skewed distribution.

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