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# Weighted jackknife ordinary kriging - problem solution of the precision in mineral resources estimation

To cite this article: W S Bargawa 2018 *IOP Conf. Ser.: Earth Environ. Sci.* **212** 012059

# Weighted jackknife ordinary kriging - problem solution of the precision in mineral resources estimation

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**Abstract.** The aim of introducing the weighted jackknife ordinary kriging (WJOK), as a robust estimator, is to solve the outliers and the problems of uncertainty associated with grade estimates of mineral resources. Jackknifing the ordinary kriging (OK) estimates and weighting the pseudo-value yield more reliable estimates than ordinary kriging and jackknife ordinary kriging (JOK) estimates. The new estimator is more conservative than the standard jackknife technique, which is shown by a little standard deviation value. It obtains a simple way to calculate the confidence interval associated with the estimation.

## 1. Introduction

One successful work of exploration is the accuracy of estimate grade of mineral resources which will be used as reference data for the next phase of work. The estimates grade, reliable, is only possible if the application of valuation technique is used according to the geological conditions of the mineral deposits [1, 2]. Complex mineralized structure may cause the appearance of high grade outliers so as to generate a highly skewed distribution of mineral grade [3-6]. This problem causes difficulty in determining the grade estimation technique. Many approaches have been carried out to solve the problem of outliers but did not yield the expected results. Moreover another problem is that the proposed approaches do not produce a measure of uncertainty of grade estimation.

This paper uses a technique to estimate the grade of mineral resources namely WJOK technique. This technique is a development of jackknife standard technique [7, 8]. Jackknife technique was previously only used to reduce bias [9-11], determine the weight [12-13], build confidence interval [14-16], further develop the application in the field of banking and financial aspects [17], and other applications [18-20]. This paper will combine classical statistics with independent data and spatial statistics with corresponding data, so that it becomes an interesting new approach. The advantage of this technique is simplicity in the mathematical concept but robust in the application on abnormal situations. This technique is more easily understood and used by practitioners in the field.

## 2. Objective

The aim of this paper is to implement a robust and reliable estimator in mineral resources which have high-grade outliers and have a long tail distribution and compare the new technique with other estimators for measuring the uncertainty of estimation generated by the standard deviation.

## 3. Method and material

The method of estimation in this paper is a combination of weighted jackknife resampling techniques [21] and ordinary kriging technique [22, 23] called jackknifing spatial data. The term of the spatial data here is ordinary kriging grade estimates.

### *3.1. Jackknife technique in linear models*

The basic concept of resampling techniques [7] can be studied through a linear model. The model assumed reads as follows:

$$Y = A\beta + e \tag{1}$$

Assume

$$D_0 = A^T A \tag{2}$$

then the least squares estimator:

$$\hat{\beta} = D_0^{-1} A^T Y \tag{3}$$

and the residual vector is:

$$R = Y - A\hat{\beta} = \left(I - AD_0^{-1}A^T\right)Y \tag{4}$$

Based on the above equation weights can be defined as follows:

$$w_i = x_i^T D_0^{-1} x_i, \qquad i = 1,\dots,n \tag{5}$$

The procedure of calculation standard jackknife technique initiated by defining pseudovalue:

$$P_i = n\hat{\beta} - (n-1)\hat{\beta}_{-i} \qquad (i = 1,\dots,n) \tag{6}$$

Jackknife standard estimator obtained from equation (6), namely:

$$\tilde{\beta} = n^{-1}\sum P_i \tag{7}$$

Cressie [24] suggests this approach is often inaccurate when applied to the unbalanced models. This weakness is reflected in the distance $w_i$. Pseudo value $P_i$ (equation 6) is defined symmetrically, while the model is usually an unbalanced model. Based on the problems this paper proposes a weighted pseudo value approach:

$$Q_i = \hat{\beta} + n(1 - w_i)\left(\hat{\beta} - \hat{\beta}_{-i}\right) = \hat{\beta} + nD_0^{-1} x_i R_i \tag{8}$$

Jackknife weighted estimator is:

$$\tilde{\beta}_w = n^{-1}\sum Q_i = \hat{\beta} \tag{9}$$

and variance estimates are:

$$V_w = \{n(n-p)\}^{-1}\sum\left(Q_i - \tilde{\beta}_w\right)\left(Q_i - \tilde{\beta}_w\right)^T \tag{10}$$

Jackknife approach is very interesting, especially if the estimates result is not only an arithmetic mean, but also a weighted average [21]. The weights are determined based on the spatial correlation between samples.

### *3.2. Kriging technique*

OK is a very popular technique because it is widely used for grade estimation in precious metals [25-27]. OK estimator is defined:

$$Z^* = \sum_{i=1}^{n} w_i Z_i \tag{11}$$

Weight, $w_i$, solved by OK equation system namely:

$$\sum_{i=1}^{n} w_j.\sigma_{ij} - \mu \;\; = \sigma_{0i} \tag{12}$$

with the constrained condition:

$$\sum_i w_i = 1 \tag{13}$$

OK equation system above can be written in matrix form as follow:

$$[D]\,[w]=[X] \tag{14}$$

Weights solved using equation:

$$[w]=[D]^{-1}[X] \tag{15}$$

Equation (15) is similar to the equation (5) so that the OK estimates can be identified with a regression line in the linear model. Based on these arguments, OK can be understood as a linear model in the case of spatial statistics. The combined concepts of spatial statistics namely OK techniques and jackknife resampling technique is a good concept and very useful to solve the unbalance model.

### *3.3. Weighted jackknife ordinary kriging*

Based on the above discussion an approach is obtained that is WJOK. In mathematical terms it can be explained as follows: sample $z_1, z_2, \ldots, z_n$ as n representing the grade data, which is independently and identically distributed (iid) at a location $x_1, x_2, \ldots, x_n$. $Z^*(x_0)$ abbreviated $Z^*$ is an estimate of grades (e.g. ordinary kriging estimate) at the location $x_0$, using n sample, and the $Z^*_{-i}$ is ordinary kriging estimate using the *n-1* sample with the *i*-th sample is not used, the jackknife estimator can be obtained by calculating the weighted pseudo value (see equation 6 and 8):

$$Zp_{-i} = Z^* + n(1-w_i)(Z^* - Z^*_{-i})\quad i=1,2,..n \tag{16}$$

Jackknife estimator is the average of weighted pseudo value [see equation (9)]:

$$Z_J = n^{-1}\sum_{i=1}^{n} Zp_{-i} \tag{17}$$

Pseudo value formulation provides an important contribution that the precision of jackknife estimator can be known through the variance or standard deviation of the WJOK, which are defined (see equation 10) as follows:

$$\sigma_J = \left[\{n(n-1)\}^{-1}\sum_i (Zp_{-i} - Z_J)^2\right]^{1/2} \tag{18}$$

Based on Tukey [7] *n* pseudo value in equation (14), can be treated as an independent estimates. Based on the above discussion, the statistics:

$$n^{1/2}(Z_J - Z)/\left\{(n-1^{-1})\sum_i (Zp_{-i} - Z_J)^2\right\}^{1/2}\quad i=1,2,..,n \tag{19}$$

will have distribution *t* with the *n*-1 degrees of freedom. Where *n* is very large the distribution will be close to normal, so it can be used in the estimation of the confidence interval [28]. Confidence interval proposed by Tukey is as follows:

$$Z_J \pm t_{\alpha,n-1}\sigma_J \tag{20}$$

with (1-2$\alpha$), 100% confidence interval for Z. In this case *t*, *n*-1 is the number of top percentile of distribution-*t* with *n*-1 degrees of freedom.

### *3.4. Grade estimation and size precision*

The main issue in grade estimation is the size precision of the estimates. The misunderstanding for kriging variance is considered the size precision of the kriging estimate. Whereas OK variance or standard deviation is not directly related to the data. The OK standard deviation is just a function of the data configuration. OK standard deviation decreases with an increasing number of samples used to estimate grade and distribution of samples around the block to be estimated. While the standard deviation of JOK reflects the actual uncertainty from the process of grade estimation. If the samples around the block to be estimated have a similar grade value, the uncertainty associated with the estimated block grade will be low in accordance with the lower value of the standard deviation of jackknife ordinary kriging. For blocks surrounded by grade sample with a mixture of high and low, the uncertainty associated with the assessment grade.

Based on the discussion jackknife ordinary kriging standard deviation is an useful indicator to quantify the uncertainty associated with the block grade estimates. Nevertheless the standard deviation of every block does not offer many practical benefits. Therefore the standard deviation value is not independent statistically. And it will be increased, which is reflected by the increasing value of the standard deviation of JOK within these blocks, there is no easy way to use it. The urgent need encountered in the estimation of mineral reserves is a measure of global estimate precision for a common form that is much larger than the block.

JOK technique allows an easy way to solve the problem. The approach introduced in this paper is block kriging short cut [29] and followed by jackknife technique. Grade estimation process is performed locally by ordinary block kriging techniques. The weights received by each sample are collected and totaled. The weight of each sample is divided by the total weight of the entire block to produce normal weight for each sample. The weights reflect the effect of each sample against global grade estimate. The term normal weight used for the sum of weights for all samples is one. Jackknifing global estimates with each remove a sample of the data can produce jackknife estimates and standard deviation estimates. The standard deviation can be used to calculate the confidence interval for jackknife estimate.

The case study conducted by reviewing nine blocks (2D illustrations for ease of explanation) which continued to be a common shape (see figure 1 below).

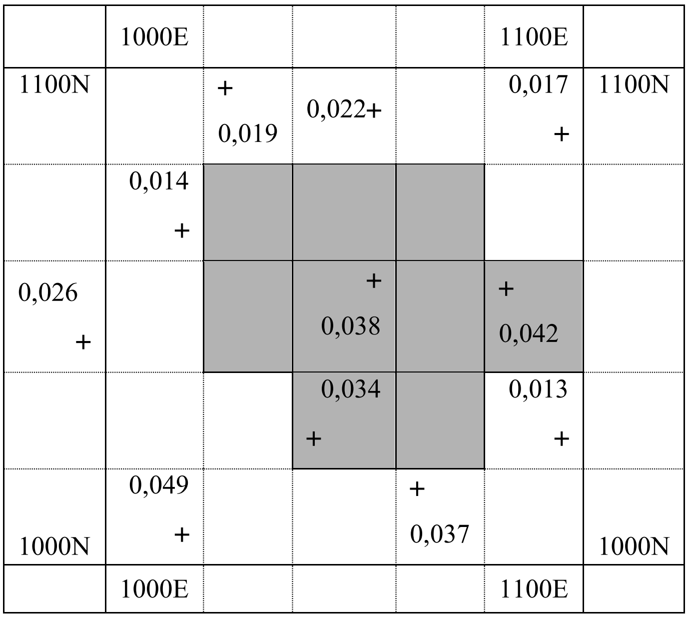


**Figure 1.** The general shape consisting of nine blocks and eleven sample grades for estimation.

| | | | | | | |
|---|---|---|---|---|---|---|
| | 1000E | | | | 1100E | |
| 1100N | | | | | | 1100N |
| | | 0,022 | 0,029 | 0,032 | | |
| | | 0,029 | 0,035 | 0,036 | 0,034 | |
| | | | 0,035 | 0,032 | | |
| 1000N | | | | | | 1000N |
| | 1000E | | | | 1100E | |

**Figure 2.** Estimates of block grades located in any shape using OK.

Grade and standard deviation of the common shape will be estimated by block kriging shortcut (OK) and jackknife. Based on the variography study of the data set used [8] in this case study can be obtained semivariogram parameters as follows: C0 (nugget) = 0.0005; C1 = 0.0045 and range = 33 m. Search radius of the block estimated sample was 16.5 m.

Practically the search of samples used for estimation was performed per block as in ordinary block kriging. Figure 2 shows the estimation using OK techniques, while Figure 3 shows the estimates of JOK for the same blocks. OK estimate average of nine blocks in Figure 2 is 0.032 oz/t, while the average of the JOK is 0.033 oz/t (figure 3).

OK estimation of the same shape using block kriging shortcut equal to the average grade of the nine blocks in figure 2 is 0.032 oz/t. Furthermore, OK estimation using block kriging shortcuts with the first example is not used for the estimation. Furthermore, each sample *i* removed from the data (*i* = 1, 2, ..., 11). Based on the calculation, it obtained pseudo value (equation 3), JOK estimates (Equation 4) and JOK standard deviation (equation 5). Global grade estimates for any form of case studies using block kriging shortcut followed by jackknife technique is 0.033 oz/t. These results are similar to the average grade of the nine blocks in figure 3. The calculation of standard deviation JOK to estimate the global results is 0.0041 oz/t.

| | | | | | | |
|---|---|---|---|---|---|---|
| | 1000E | | | | 1100E | |
| 1100N | | | | | | 1100N |
| | | 0,020 | 0,031 | 0,033 | | |
| | | 0,028 | 0,038 | 0,043 | 0,038 | |
| | | | 0,034 | 0,033 | | |
| 1000N | | | | | | 1000N |
| | 1000E | | | | 1100E | |

**Figure 3.** Block grade estimates located in any shape using JOK.

The confidence interval for the global estimation can be calculated using equation (7). According to Student-t table, the value of $t_{\alpha, n\text{-}1}$, $\alpha = 0.025$ and $n\text{-}1 = 10$ was 2.228. If fixed at 95% confidence interval for jackknife ordinary kriging global estimation then obtained: (0.033±2.228*0.0041) oz/ton. In other words there is a 95% probability that the gold grade estimate of the volume (any shape) ranges between the value of 0.042 oz/ton and 0.024 oz/ton.

The second case study conducted in porphyry copper deposits. The amount of data that is used by 20,720 assay obtained from 207 drill data includes copper grade (assay), specific gravity, and rock type. Generally drill hole was carried out by the method of inclined drilling to the copper ore deposits. Composite data based on geological considerations with interval length correspond at 15 m mining bench height. Composite number is 6,400 data. 3D block model is made with block size of (20 x 20 x 15) m. Size of (20 x 20) m shows the north and east, while 15 m is a measure of mining bench height. Each block in the model is marked with rock code which indicates the blocks that are inside or outside the mineralized area. Interpolation is not performed on blocks that are outside the mineralization area.

Figure 4, 5, and 6 below is a scatter diagram between copper composites and block grade estimates at the same location. The figure shows the result of the estimation of block grade using OK, JOK, and WJOK methods. Linear regression statistics are shown in table 1. Statistics show a comparison between the performance of the estimator used. Visual observation of the scatter plot (figure 4, 5, and 6) clearly shows that the estimated grade using WJOK method tends to be closer to $45^0$ regression line (bisector) compared to the estimated grade using the OK and JOK method. Table 1 shows that the WJOK estimator has the lower Y-intercept value, the higher the slope value compared with OK and JOK estimator.This case study shows that the estimator WJOK is more accurate than the OK and JOK estimator.

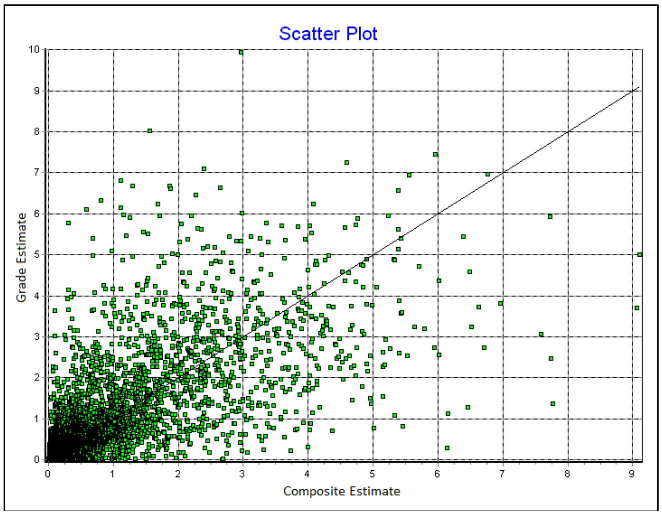


**Figure 4.** Grade estimates using OK, grade estimates were more spread.

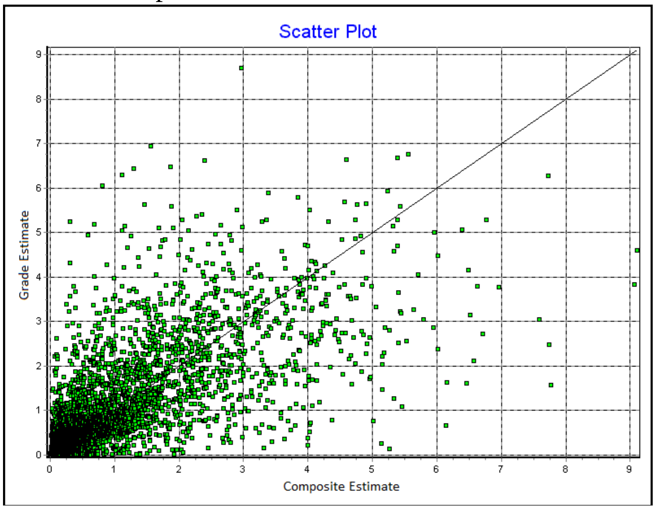


**Figure 5.** The grade estimates using the JOK method also tends to spread.

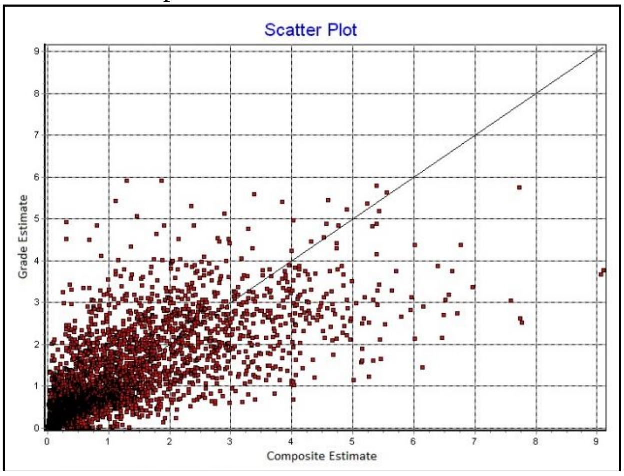


**Figure 6.** The grade estimates of the WJOK method tend to follow the $45^0$ regression line.

**Table 1.** Linear regression statistics based on the scatter plot between the copper composite and grade estimate.

| Parameter | OK | JOK | WJOK |
|---|---|---|---|
| *Y-Intercept* | 0.233 | 0.195 | 0.147 |
| *Slope* | 0.791 | 0.815 | 0.850 |
| *SEE* | 0.623 | 0.589 | 0.498 |
| *R* | 0.680 | 0.750 | 0.800 |

## 4. Discussion

Usually the addition of a number of exploration drillings in an area of investigation is conducted to produce a more detailed geological information in order to obtain accurate data. Geologist and mining exploration experts can estimate the effect of the presence or absence of drill point at a certain location. If there is no drill point at a location it will produce different geological information if the location was coupled with the drill point. The simulation of a present or an absent drilling point in one location will display a spatial image of the geological model at that location.

Jackknife resampling technique is similar to the above scenario that is how the estimate results if one particular sample of data is not used. Similarly the simulation for each sample will be removed from the data and re-used in the next estimation. Combining estimation using all samples and estimation when each sample is removed from the data can utilize all the information throughout the data. The idea is similar to the addition of information from a drilling point at specific locations and how geological information changes if the particular location does not have a drilling point.

The interesting concept is the basis for the use of WJOK techniques for ore grade estimation. The case study in this paper is used to indicate the accuracy of the estimator and measurement of uncertainty. In a case study of a bench composite with the relatively uniform grade distribution the third estimation techniques (WJOK, JOK, and OK) produce similar estimates, but the standard deviation value of the WJOK is smaller than the standard deviation of the standard jackknife OK. In this case the variance (ordinary) kriging, can not be compared with the jackknife variance because it is not a measure of uncertainty or estimates precision. During this time there is a fallacy that kriging variance is a measure of precision whereas the kriging variance is merely an index of data configuration in the estimation.

The second case study shows that the WJOK is more robust than the JOK and OK techniques. OK estimates are to be high due to the influence of outlier extrapolation. The estimate is less realistic as the blocks are estimated to be surrounded by samples that have a relatively low value. While the block grade estimates using WJOK technique indicates the most reliable results. The standard deviation value of the WJOK is smaller than the standard deviation value of the JOK. The WJOK technique is more conservative because it has a wider confidence interval than the JOK technique.

## 5. Conclusion

The discussion in the previous chapter can be summarized as follows:

1. WJOK estimator is a new technique that is more robust in the unbalanced model situation compared with JOK and OK techniques.
2. The use of repetitive samples can maximize spatial correlation information between samples in order to obtain the block grade estimates which is more accurate and can generate confidence interval estimates.
3. Combining jackknife resampling approach and spatial statistics in the form of jackknifing ordinary kriging estimates is a new approach that is highly prospective for mineral resource grade estimation.

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