Gold Resource Modeling Using Pod Indicator Kriging

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Abstract. This paper describes an implementation of the pod indicator kriging method used to gold resource modeling. Method such as ordinary kriging estimate the mean grade of a block that is fairly large. The usual outcome is that large blocks rarely turn out to be all ore or all waste, thus making reserve estimates an incorrect estimate of what will be mined. Pod indicator kriging offers a solution to this problem by estimating the distribution of grade values within a large block, rather than just estimating the mean grade of the block. Knowing the distribution of grade value within the block, it is then easy to calculate the proportion of the block that is above cutoff grade and the grade of the ore above cutoff grade. This research shows that the pod indicator kriging model is quite applicable and reliable in gold resource modeling.

INTRODUCTION

Selecting method of valuation grade of gold ore reserves becomes difficult when the encountered histogram of data with a long tail distribution (or mineralization) interspersed by waste material. Various estimator ore reserves have been introduced to deal with problems in the estimation. These estimator have not, however, enjoyed the widespread acceptability and use afforded the ordinary kriging (OK) estimator as they are difficult to comprehend and apply. If recoverable reserve estimators are to attain the level of acceptability of ordinary kriging, methods of estimating recoverable reserves must be developed which are simple, robust, and easy to apply. The application of pod indicator kriging (PIK) is presented here. The method was applied in assessing uncertainty in estimates with ordinary and indicator kriging refer to [1] and [2] and [3]. It was also used by [4] for the evaluation of arsenic potential contamination, and [5] for modeling in iron ore deposit.

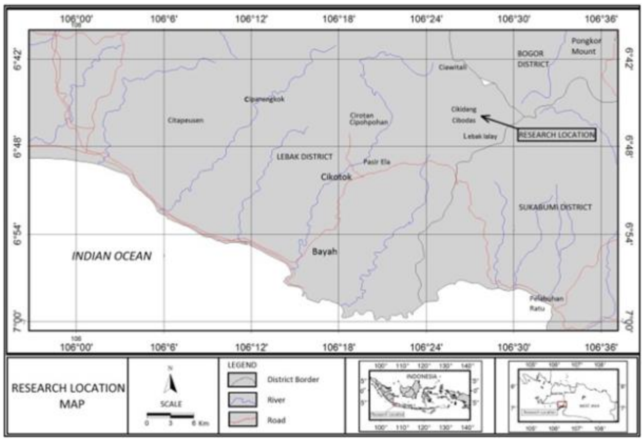

FIGURE 1. Location of research area

AIP Conf. Proc. 1705, 020025-1-120025-8; doi: 10.1063/1.4940273

This paper presents a case study illustrating the application of two estimators, PIK and OK on the Cikidang gold vein deposit. FIGURE 1 shows the Cikidang gold epithermal deposit located in Lebak District West Java Indonesia and is operated by PT Aneka Tambang. FIGURE 2 shows geological map of Cikidang area of West Java Province of Indonesia. The strike and dip of Cikidang vein respectively are N21^{0}E and 75^0 to west. Cikidang deposit is postulated as a tension gash fracture infilled by gold bearing hydrothermal fluid, forming quartz vein and altered the wall rocks. The Cikidang oblique fault with its minor faults (normal and reserve faults) was formed during the post mineralization stage, which influenced the geometry of the deposit. High grade Au-Ag mineralization is generally found in silicic alteration associated with shear zones and the mentioned clay minerals.

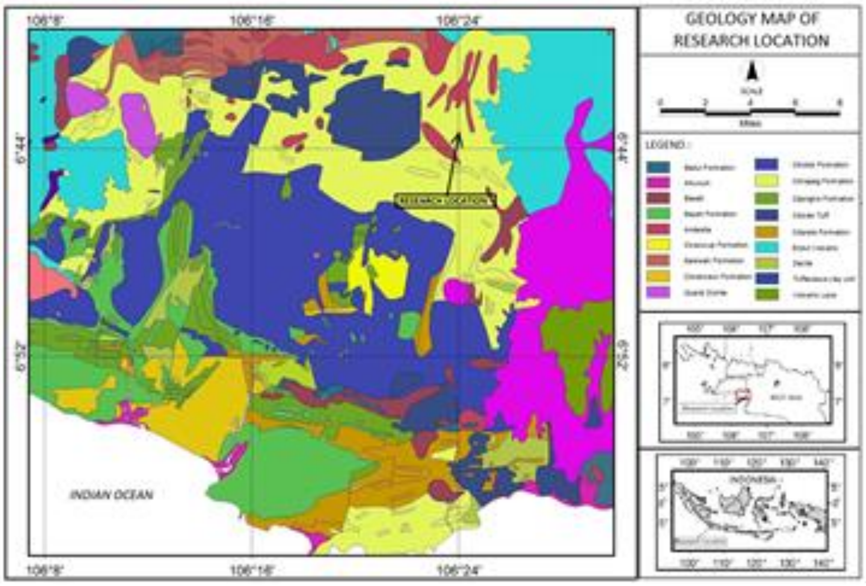

FIGURE 2. Geological map of Cikidang area West Java Province of Indonesia

OBJECTIVE

The main goals of the current work are: (**a**) to evaluate an application of geostatistical method through case study with gold resource modeling, (**b**) to analyze performances of ordinary kriging and pod indicator kriging with block model which describes 3D depiction of geometry of mineralization and grade distribution in gold resource modeling.

METHODS AND MATERIAL

Geostatistics

Geostatistics is a rule which the main element is regionalized variables that depends on the spatial location. Spatial data like the resources of gold vein deposit can be assumed as result of observations on a stochastic process as $\mathcal{Z} = Z(s) : s \in \mathcal{D}$. Z($s$) here is a random function which in practice never really known, s is a point within the spatial domain of $\mathcal{D}$. Geostatistics consist of semivariogram i.e. the main tool of spatial variables that quantified the size and intensity of spatial variation, provide the optimum interpolation or estimation through kriging technique. Cressie and Hawkins defined robustness ideas originated from the thinking that Matheron semivariogram are affected by a typical observations and transforms to Box and Cox (1964) in order to obtain formula [6]

$$\gamma(h) = \left(\frac{1}{2|N(h)|} \Sigma_{N(h)} \left(Z(s_i) - Z(s_j) \right)^{1/2} \right)^4 / \left(0.457 + \frac{0.494}{|N(h)|} \right) \tag{1}$$

Progress on Applied Mathematics in Science and Engineering
AIP Conf. Proc. 1705, 020025-1-120025-8; doi: 10.1063/1.4940273
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Kriging is the interpolation technique, which refer to spatial sampled data. This technique uses the stationary concept which is seen more stochastic because it was trying to choose the optimal weights, by minimizing the estimation of variance error. Some assumptions and simplifications in this method are, that the observation data can be seen as a realization of a random variable, which is formally presented as $Z(s); s \in \mathcal{D}$. In general, if the z(s_i) (i=1, …, n) is n observations data residing in many locations of $\mathbf{s}_i$, and $\mathbf{s}_0$ is a position of the predicted point, the prediction value, $\hat{Z}(s_0)$ can be written as [6]

$$\hat{Z}(s) = \sum_{i=1}^{n} w_i z(s_i) \tag{2}$$

is a weighted average of grade sample and weigh, w_i which $\sum_{i=1}^{n} w_i = 1$ and will be estimates is assumed as unbiased.

Pod indicator kriging (PIK) as in [7] is a simplification of multiple indicator kriging (MIK) which the objective is to the population spread based on data, if the geological interpretation incomplete or even non-existent. Mathematical approach to describe the method of IK, if the gold grade $z(s)$ at the location s, where $s \in \mathcal{D}$ or L is a deposit which is assumed to limit the levels of z_c, then every point of s can be expressed as [8]

$$i(x, z_c) = \begin{cases} 1 \ if \ z_x \geq z_c \\ 0 \ if \ z_x < z_c \end{cases} \tag{3}$$

PIK philosophy stems from the indicator kriging conception (IK), which is a technique that although the assessment takes into account the levels remain high (or outlier levels) but not use the normal distribution assumption. Therefore, this estimator can produce the confidence interval on the valuation reserves. If P_0^* are required probability, w_i is the weight and I_i is the value of the indicator data at the point i then

$$iP_0^* = \sum_{i=1}^{n} w_i I_i \tag{4}$$

Based on the equation (4), all the data is then converted into the new variable, L, which the value is 0 and 1. Proportion (P) of the variable $z(s)$ below the cutoff level z_c in area $A \subset D$ is expressed in the equation bellow

$$P(A;z) = \frac{1}{A}\int_A i(x;z_c)dx \in [0,1] \tag{5}$$

If P (A; z) is known, the proportion of variable above the cutoff level expressed z_c

$$proportion \quad z(x) > z_c = 1 - P(A;z) \tag{6}$$

$i(x;z)$ and $P(A; z)$ as a function of z can be seen as the CDF (cumulative distribution function) with $P(A; z)$ be the average for all $s \in A$. The proportion can be estimated with n data values of $z(s)$. If $i(s_\alpha)$ with $s_\alpha \in A, \alpha=1,\ldots,n$ can be expressed as

$$P^*(A;z) = \frac{1}{n}\sum_{\alpha=1}^{n} i(s_a; z_c) \tag{7}$$

It is the simple arithmetic average of the indicator variables.

Based on the continuity of space $z(s)$, $\{z(s), s_\alpha \in \mathcal{D}$ can be obtained estimates of

$$P^*(A;z) = \frac{1}{n}\sum_{\alpha=1}^{n} \lambda_a(z_c).i(s_\alpha : z) \tag{8}$$

with $s_\alpha \in \mathcal{D}$ and n weight of $\lambda_\alpha(z_c)$.

Block model construction

The 3D depiction of geometry of mineralization, ore grade distribution, and the resources in gold vein deposit can be obtained using block model. The first step in geologic modeling is to plot cross sections with geologic data of each drill hole on sections. The geology is then interpreted on the cross section and a numerical code is assigned to each rock type to represent the geology in digital form for computer input. Geology from the section is then plotted onto horizontal plan maps. It shows where each cross section intersects the plan map and the rock types at the intersection. After the geologic modeling is complete, plan maps of the model can be plotted and checked. If a computer block model is to be used to model an orebody that is to be mined by open pit methods, it is important to enter topography into the model.

The limits of the ore reserve model are plotted on a topographic map of the area. Topographic contour are digitized to obtain several thousand data point with the northing, easting, and elevation of each point. Gold resource modeling in this study have coordinate boundary as follow: North direction: 0 N – 1,300 N; East direction: 150 E – 600 E; Elevation: 1,075m – 1,400m.

RESULT

Construction of exploration database consists of making assay and composite database descriptive statistics, and variography study. There are 57 core drill holes with average spacing of 75m. A statistical summary of those is given in TABLE 1. As can be seen, the assays are moderately skewed to the right with a coefficient of variation of 1.50.

Statistical analysis and geological interpretation indicate that the data and blocks in the model could be divided into two populations, i.e. vein and non-vein. Decile analysis of the data show that high-grade cap is not required. To obtain the same geometrical support, assay data are averaged into 2 m composites. Composite statistics give mean grade of 12.80 g/t Au, as shown in TABLE 2.

TABLE 1. Statistical summary of Au assays

Statistics	Gold assay
Number of samples	681
Maximum (g/t)	104.37
Minimum (g/t)	0.02
Mean (g/t)	9.71
Median (g/t)	3.91
Standard deviation (g/t)	14.28
Variance $(g/t)^2$	203.99
Skewness	2.68
Kurtosis	12.04
Coefficient of variation	1.50

TABLE 1 and TABLE 2 show that composite grade mean is higher than assay grade mean, because approximately 60% data are taken from high grade vein area with 2 m interval sampling.

TABLE 2. The result of Au-Ag grade composite statistics

Statistics	Gold composite
Number of data	498.00
Maximum (g/t)	91.10
Minimum (g/t)	0.02
Mean (g/t)	12.80
Median (g/t)	8.10
Standard deviation (g/t)	15.30
Variance $(g/t)^2$	233.16
Skewness	2.26

Variogram modeling

Searching data to along strike of vein (N21^{0}E) and down dip of vein (75^0 to West) of research area obtain variography model. Along strike and down dip gold variograms are calculated using all data. These variograms are calculated through covariances. It is generally more difficult to obtain traditional variograms for gold deposits, and the Cikidang vein deposit is no exception. As can be seen from these figures, all variograms are quite well behaved and they can be fitted with the same spherical models.

TABLE 3. PIK and OK estimation parameters

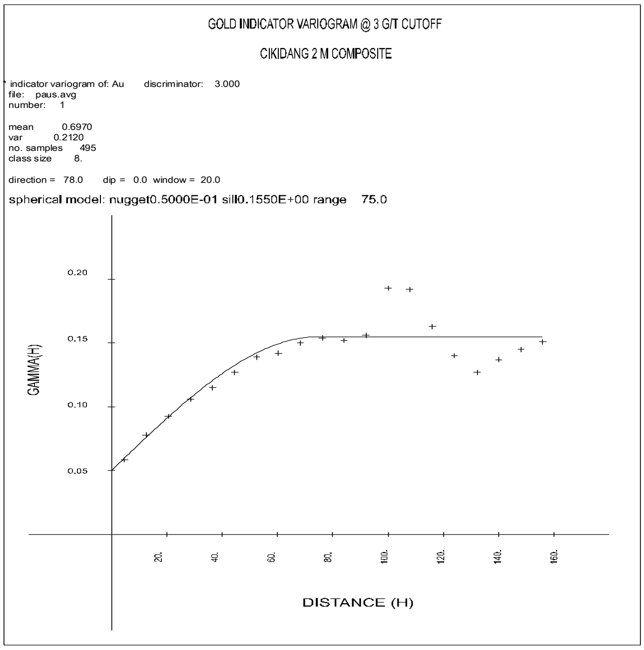
No.	Parameter	PIK	OK
1.	Search distance: along strike of vein	50 m	45 m
2.	Search distance: down dip of vein	35 m	40 m
3.	Search distance perpendicular to vein	1.50 m	5 m
4.	Minimum number of sample	2	2
5.	Maximum number of sample	10	10
6.	Variogram or Indicator variogram:		
	• nugget	0.050	100 $(g/t)^2$
	• sill	0.155	8,000 $(g/t)^2$
	• range	75 m	45 m
7.	Geological control: Grade estimation is only done in vein area		

Along strike and down dip indicator variograms are calculated at one discriminator grade, i.e. 3 g/t Au. This first indicator variogram is used to define blocks, which exist within mineralization area (up to 3 g/t Au). FIGURE 3 show the resulting indicator variograms.

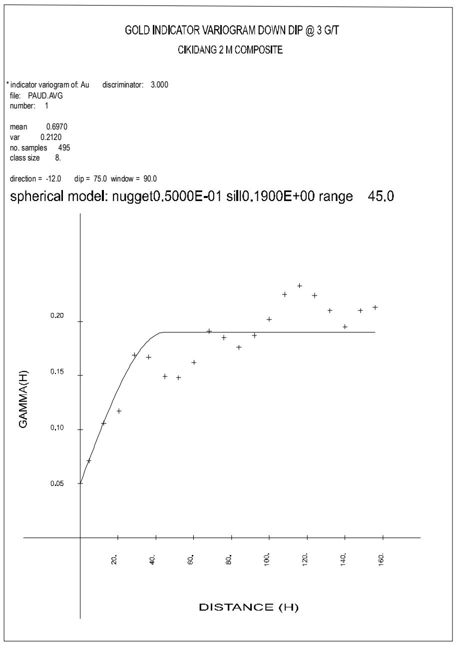
The variogram is also quite well behaved and it can be fitted with spherical model as well. Gold grade estimation parameters, which are obtained from ordinary and indicator variograms and data distribution, are shown in TABLE 3.

Pod indicator kriging

Reserves estimation for the gold vein deposit boils down into two steps. First, the amount of ore within a volume must be determined and second, the grade of the ore must be determined. The process is called pod indicator kriging. Pod indicator kriging involves two steps. First, a study of the gold vein deposit is done to determine a cutoff value, i.e. 3 g/t Au that clearly separates mineralized material from non-mineralized ground. Next, a variogram is calculated for the indicator values. Using the indicator variogram, ordinary kriging is then used to estimate the propotion of each block that is waste. It should be obvious that if we know the percentage of each block that is waste, we also know the amout of each block that is ore. The second step is to determine the grade of the ore within each block. The grade of the ore in the block should be correlated to the grade of ore observed in nearby samples. Thus, the second step is to estimate the grade of the ore using only nearby samples that are ore grade. Ordinary kriging is used to make these estimates. The variogram used for the kriging correspond to the variogram calculated from data that are above the cutoff grade.



(a)



(b)

FIGURE 3. Gold indicator variogram along strike (a) and down dip (b) @3g/t Au

Gold resource modeling in this study use PIK technique with OK technique for comparison. The OK estimation parameters for block estimation are roughly similar to those of the PIK model. The PIK model used only a 3 g/t Au discriminator to define the mineralized vein. Table 4 shows the result of the gold vein modeling using PIK and OK techniques. Based on the research the section of OK block model is controled by ore vein that is obtained from geological digitized, whereas the vein shape of PIK model reflects the actual condition. The PIK model could distinguish high grade area and low grade area. The comparison of tonnage, grade, and gold metal content of PIK and OK models using three composites minimum can be seen at TABLE 4. Both of PIK and OK models show high grade estimates because they use drilling data and stope sampling in the vein area.

TABLE 4. Tonnage, grade and gold metal content of PIK and OK models

Cog (g/t)	PIK model (tons)	OK model (tons)
0	404,169	821,517
4	400,569	623,949
6	376,274	529,883
10	295,729	376,286
Cog (g/t)	**Grade (g/t) of PIK model**	**Grade (g/t) of OK model**
0	18.62	12.32
4	18.76	15.55
6	19.59	17.40
10	22.66	21.32
Cog (g/t)	**Au metal content of PIK model (kg)**	**Au metal content of OK model (kg)**
0	7,525.63	10,121.09
4	7,514.67	9,702.41
6	7,371,21	9,219.96
10	6,701.22	8,022.42

DISCUSSION

To compare the performance of both estimators is performed as follows: the distribution of samples used in grade estimation is plotted against the distribution of block estimates at sample locations. The cumulative frequency plot of the PIK and OK estimates, which are plotted against the data value, show that in the low-grade range, the estimate is plotted above the composite grade, which indicates over estimation. Similarly, in the high grade range the block estimate is lower than the composite grade. This is a common feature found in kriged estimates (referred to as smoothing) especially where one is estimating grade in a block volume, which is much larger than the sample. However, the two distributions are quite close to each other.

The ideal scatter plot between the data value and its estimate is the first bisector, i.e. the 45-degree line with zero intercept. The scatter plot shows the PIK and OK estimates are plotted against the data value, this means every single estimate matches the data value perfectly at all locations. In the real world this is, of course, unattainable.

TABLE 5 shows the linear regression statistics for each scatter plot. Comparison of both PIK and OK models indicate that the PIK technique is quite reliable in gold vein modeling.

TABLE 5. Linear regression statistics: scatter plot of composite grade against estimated grade

	PIK	**OK**
Number of samples	255	235
Intercept	3.02	4.22
Slope	0.65	0.73
Standard error of estimate (SEE)	2.36	5.90
Coefficient of correlation (R)	0.70	0.60

Gold vein deposit, in this study, is characterized by a low average concentration, high variance, and a skewed distribution of assay values which tends to produce outliers, poor continuity of mineralization, complex intermingling of ore and waste, and difficulties in obtaining reliable samples. Each of these factors adversely affects the reliability of ore reserve estimation. PIK could help to solve several of the above problems. Outliers make interpretation of variogram difficult, thus lending suspicions to the validity of OK results. With PIK, variogram are calculated using indicator value (0 or 1). The variogram calculation is thus unaffected by outliers.

The poor continuity and often complex intermingling of ore and waste mean that precious metal deposits usually have to be mined in a highly selective manner. This causes problems in trying to contruct an ore reserve model from wide spaced exploration data that will correctly estimate the minable reserves when the deposit is actually mined in a selective manner. OK method estimate the mean grade of a block that is fairly large. However, when the deposit is mined, selection is based on a smaller volume, called the selective mining unit (SMU). The usual outcome is that large blocks rarely turn out to be all ore or all waste, thus making our reserve estimate an incorrect estimate of what will be mined. PIK obtains a solution by estimating the mean grade of the block.

CONCLUSION

The advantage of PIK is gold resource modeling does not require digitization of the geological model, but the PIK can define ore based selection discriminator value to determine the area of mineralization. Grade estimation is only done on the mineralization area. PIK model have relatively high grade mean than OK model, because low grade have screened with determination a 3 g/t Au discriminator. Au metal content of PIK model is relatively conservative than OK model.

REFERENCES

1. C.D. Lloyd and P.M. Atkinson, Assessing uncertainty in estimates with ordinary and indicator kriging, *Journal of Geocomputation & Geosciences* **27**, 929 – 937 (2001).
2. O. Marinoni, Improving geological models using a combined ordinary–indicator kriging approach, *Journal of Engineering Geology* **69**, 37 - 45 (2003).

3. I.M.H.R. Antunes and M.T.D. Albuquerque, Using indicator kriging for the evaluation of arsenic potential contamination in an abandoned mining area (Portugal), *Journal of Science of The Total Environment* B, 545–552 (2013).
4. B. Mehdi et al, The application of median indicator kriging and neural network in modeling mixed population in an iron ore deposit, *Journal of Computers & Geosciences* **37**, 530 – 540 (2011).
5. R. Webster and M.A. Oliver, *Geostatistics for environmental scientists* (Second Edition. England: John Wiley & Sons Ltd., 2007).
6. N. Cressie, *Statistics for spatial data* (John Wiley & Sons, Inc, New York, Chichester, Toronto, Brisbane, Singapore, 1993), pp. 281-283.
7. F.V. Meera, Indicator kriging applied to absorption band analysis in hyperspectral imagery: A case study from the Rodalquilar epithermal gold mining area, SE Spain, *Journal of Applied Earth Observation and Geoinformation* **8**, 61 – 72 (2006).
8. A.G. Journel, Non Parametric Estimation of Spatial Distribution, *Journal of Mathematical Geology* **15**, 445-468 (1983).
9. E.J. Hill et al, Modelling ore bodies of high-nugget gold using conditional probability, 15th Annual Conference Proceeding 2014, *The International Association for Mathematical Geosciences*, pp. 5-8.

AIP Conf. Proc. 1705, 020025-1-120025-8; doi: 10.1063/1.4940273