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ECONOMIC ORDERING POLICY OF DETERIORATING ITEM WITH INCREMENTAL DISCOUNT UNDER PERMISSIBLE DELAY IN PAYMENTS

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ABSTRACT
In the classical Wilson's economic ordering quantity model, it was assumed that the goods can be held infinitely for future demand. However, in practices, as most items we deal with such goods required considering the factors of short shelf life, then some suppliers tries to reduce their lost by offering incremental quantity discount. This paper will try to combine and modify several basic models of deterministic inventory control to design a model for deteriorating item by considering quantity incremental discount under permissible delay in payments. Heuristic approachis used to solve the models. A numerical example has been solved to illustrate the model.

Keywords: EOQ, deteriorating item, incremental discount, permissible delay in payments.

INTRODUCTION
Deteriorating items constitute a large portion of retailer inventory and lose value with time due to deterioration and/or obsolescence. The 2005 National Supermarket Shrink Survey reported that deteriorating account for more than 54% of total store sales which constitute more than $200billionand approximately 57% of total store shrink [1]. Further, deterioration become the main operating key to achieve and sustain competitive advantages. Roughly 10% of all deteriorate items (fresh products and other food products) goes to waste before consumers purchase it [2]. Thus, retailer are faced to an important dilemma offering to customers what they want so that they can achieve a higher customer service level or reducing losses by decreasing quantities on shelves which leads to frequent stock outs. Clearly, the ability to satisfy customer while reducing losses needs the application of good inventory management principles [3].

According to [4], deteriorating items refers to the items that become decayed, damaged, evaporative, expired, invalid, devaluation and so on through time. According to the definition, deteriorating items can be classified into two categories. The first category refers to the items that become decayed, damaged, evaporative, or expired through time, like meat, vegetables, fruit, medicine, flowers, film and so on; the other category refers to the items that lose part or total value through time because of new technology or the introduction of alternatives, like computer chips, mobile phones, fashion and seasonal goods, and so on. Both of the two categories have the characteristic of short life cycle. For the first category, the items have a short natural life cycle. After a specific period (such as durability), the natural attributes of the items will change and then lose useable value and economic value; for the second category, the items have a short market life cycle. After a period of popularity in the market, the items lose the original economic value due to the changes in consumer preference, product upgrading and other reasons [5].

In the classical economic ordering quantity model, it is assumed that the goods can be held infinitely for future demand. However, in practices, such an assumption is not true for a large wide of goods characterized by a limited lifetime. Retailers dealing with such goods required considering the factors of short shelf life in determining optimal procurement policy.

Other assumption is the price of the goods ordered are fixed and not dependent on the amount of goods ordered. In fact, the most common practical method for the supplier to stimulate demand, increase market share and cash flow while reducing its inventory of specific items is to offer a temporary price discount to retailers. When the supplier offers a temporary price discount for any of the above reasons, it is important for the retailer to determine whether or not it is advantageous to place a special forward buying order, i.e., purchase additional stock at the reduced price offered by the supplier for a delayed sale at the regular selling price [6].

In addition to the above assumptions, the classical economic ordering quantity model also assuming that the supplier must be paid for the items as soon as the items are received. However, in practice a supplier will allow a certain fixed period for settling the amount owed to him for the items supplied. Usually there is no charge if the outstanding amount is settled within the permitted fixed settlement period. Beyond this period, interest is charged. When a supplier allows a fixed time period for settling the account, he is actually giving his customer a loan without interest during this period. During the period before the account has to be settled, the customer can sell the items and continue to accumulate revenue and earn interest instead of paying off the overdraft which is necessary if the supplier requires settlement of the account immediately after replenishment. Therefore, it makes economic sense for the customer to delay the settlement of

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the replenishment account up to the last moment of the permissible period allowed by the supplier [7]

Research on inventory control for items that may have expired are carried out intensively by some researchers. Problem of raw material inventory that considers factors expiry studied in [8]. This model consider the raw materials that have expired can still be sold even with a very low price. Paper [9] developed a deterministic inventory model for products that have a shelf-life by considering product return. Where shortages are allowed by using backorder that equal to the amount of expired products. Research inventory taking into account the expiry factor and unit prices are also carried out by [10]. The third models still assumes that the retailer's capital is unconstrained and the retailer must be paid for the items as soon as the items were received. However, the supplier may offer there tailer a delay period, that is the trade credit period, in settling the accounts.[7] developed an EOQ model under the condition of permissible delay in payments. Then [11] developed optimal ordering policy for deteriorating items under the delay in payments in demand declining market. At this model, the demand of a product is assumed to be decreasing with time, shortages are not allowed and replenishment rate is infinite, and the retailer generates revenue on unit selling price which is necessarily higher than the unit purchase cost.

This paper will try to combine and modify several basic models of deterministic inventory control to design a model for deteriorating item by considering incremental quantity discount [12] under permissible delay in payments [7] and shortages are allowed by using backorder that equal to the amount of expired products.

The paper is divided into four sections. The first contains an introduction, the second part contains the inventory model development. Then the third and final section is a numerical example to illustrate the model and conclusions of the research.

MATHEMATICAL FORMULATION

The notations used are as the follows:

D = annual demand
Q = quantity order, unit
Q_{kd} = quantity deterioration, unit
A = cost per order
h = unit stock-holding cost per item per year excluding interest charges
Ic = interest charges per \$ investment in stocks per year
Id = interest which can be earned per \$ in a year
P = unit purchase price in \$
S = cost of placing one order
t = permissible delay in settling accounts
T = time interval between successive orders
π = cost of stockout in \$ per unit
t_1 = time interval between ordered received to ordered lossed
t_2 = time interval during stockout
S = profit value in \$ per unit
E_i = extra purchase cost for not purchasing each of Q units at Pi
U = price break
B_1 = cost of interest charges per \$ investment in stocks per year
B_2 = cost of interest which can be earned per \$ in a year
PC = purchase cost in \$ per year
OC = ordering cost in \$ per year
HC = holding cost in \$ per year
SC = shortage cost in \$ per year
EC = expired cost in \$ per year
TC = total annual variable cost

The following assumptions are made in deriving the model;

(1) The demand for the item is constant with time.
(2) Shortages are allowed by using backorder that equal to the amount of expired products
(3) During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account. At the end of this period, the account is settled and we start paying for the interest charges on the items in stock.
(4) Time period is infinite.
(5) Lead time constant
(6) Expiration date known
(7) Returns are not permitted by the supplier. Products that have expired will be destroyed
(8) The case is $T \geq t$

Model in this paper consider incremental discount factor, deteriorated item, and delays in payments where the products that have expired are immediately destroyed. It is expected to determine the optimal order quantity of products, the quantity of expired products were destroyed, and when order it, so that the total annual variable cost is minimal. The total annual variable cost consists of the following elements:

- Cost of purchase in a year

$$PC = \left(\frac{E_i}{Q} + P_i\right).D \quad (1)$$

- Cost of placing orders in a year

$$OC = A\frac{D}{Q} \quad (2)$$

- Cost of stock holding (excluding interest charges)

$$HC = \frac{\left(\frac{E_i}{Q}+P_i\right) I.(Q-Q_{kd})^2}{2Q} \quad (3)$$

- Cost of stockout by using backorder in a year

$$SC = \frac{\pi . {Q_{kd}}^2}{2Q} \quad (4)$$

- Cost of expired in a year

$$EC = (S.Q_{kd}).\frac{D}{Q} \quad (5)$$

- Cost of interest charges per $ investment in stocks per year

According to [7] that when items are sold, and before the replenishment account is settled, the sales revenue is used to earn interest. When the replenishment account is settled, the situation is reversed, and effectively the items still in stock have to be financed at interest rate I, The stock level at the time of settling the replenishment account equals D(T - t) (see Figure-2) and the interest is payable during time (T - t).

$$\text{Interest payable in one cycle} = \frac{DP(T-t)^2.I_c}{2T} \quad (6)$$

Cost of interest charges per $ investment in stocks per year

$$B_1 = \frac{DPTI_c}{2} + \frac{DPt^2 I_c}{2T} - DPtI_c \quad (7)$$

If $T = Q/D$,

$$\text{If } T = Q/D, \quad B_1 = \frac{DP\frac{Q}{D}I_c}{2} + \frac{DPt^2 I_c}{2\frac{Q}{D}} - DPtI_c \quad (8)$$

Cost of interest which can be earned per $ in a year

Interest earned during the permissible settlement period. The maximum accumulated amount earning interest during the settlement period equals Dtp if T > t (see Figure-2). Hence the cost of interest earned during the permissible settlement period is obtained as

$$B_2 = \frac{PDt^2 I_d}{2T} \quad (9)$$

$$B_2 = \frac{PDt^2 I_d}{2\frac{Q}{D}} \quad (10)$$

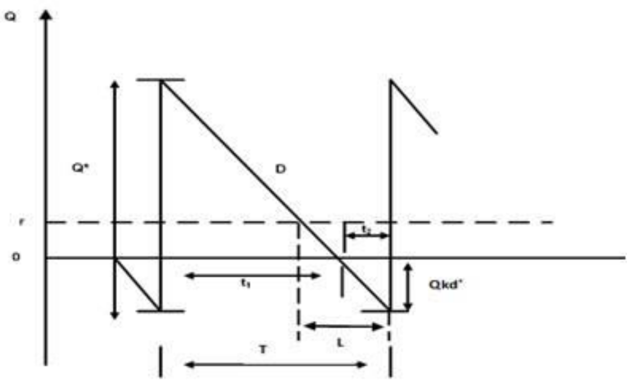

Figure-1. Backordering inventory model (Tersine, 1994).

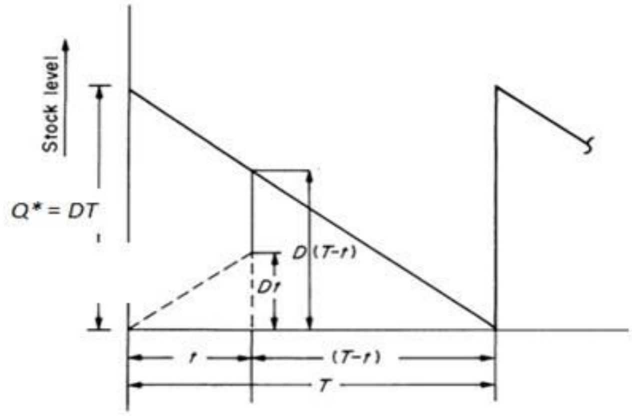

Figure-2. Typical inventory sycleunder permissible delay in payments,$T \geq t$ case [7].

The annual total variable cost are

$$TC = PC + OC + HC + SC + EC + B_1 - B_2 \quad (11)$$

$$TC(Q) = \left\{\left(\frac{E_i}{Q} + P_i\right)D\right\} + \left\{A.\frac{D}{Q}\right\} + \left\{\frac{\left(\frac{E_i}{Q}+P_i\right)I.(Q-Q_{kd})^2}{2Q}\right\} + \left\{\frac{\pi.Q_{kd}^2}{2Q}\right\} + \left\{(S.Q_{kd})\frac{D}{Q}\right\} + \left\{\frac{DP\frac{Q}{D}I_c}{2} + \frac{DPt^2 I_c}{2\frac{Q}{D}} - DPtI_c\right\} - \left\{\frac{PDt^2 I_d}{2\frac{Q}{D}}\right\} \quad (12)$$

$$TC(T) = \left\{\left(\frac{E_i}{DT} + P_i\right)D\right\} + \left\{A.\frac{D}{DT}\right\} + \left\{\left(\frac{IE_i}{2(DT)^2} + \frac{P_i I}{2DT}\right)(DT - Q_{kd})^2\right\} + \left\{\frac{\pi.Q_{kd}^2}{2DT}\right\} + \left\{(S.Q_{kd})\frac{D}{DT}\right\} + \left\{\frac{DP_i TI_c}{2} + \frac{DP_i t^2 I_c}{2T} - DP_i tI_c\right\} - \left\{\frac{P_i Dt^2 I_d}{2T}\right\} \quad (13)$$

In order to minimize TC, the optimal value of Q, Q_{kd} and T are obtained by $\frac{\partial TC(Q)}{\partial Q} = 0$, $\frac{\partial TC(Q)}{\partial Q_{kd}} = 0$ and $\frac{\partial TC(T)}{\partial T} = 0$.

$$(Q^*)^3(-P_iI + 2P_iI + P_iI_c) + (Q^*)^2(2IE_i + 2Q_{kd}{}^*IP_i + 2IE_i - 2Q_{kd}{}^*IP_i) + (Q^*)(-2DE_i - 2AD - 2SQ_{kd}{}^*D - \pi(Q_{kd}{}^*)^2 - D^2P_it^2I_c + D^2P_it^2I_d + 4Q_{kd}{}^*IE_i - P_iI(Q_{kd}{}^*)^2 - 2Q_{kd}{}^*IE_i) = 2Q_{kd}{}^2IE_i \quad (14)$$

$$Q_{kd}{}^* = \frac{Q^*IE_i + (Q^*)^2P_iI - SDQ^*}{IE_i + P_iIQ^* + \pi Q^*} \quad (15)$$

$$(T^*)^3(D^3P_iI - 2D^3P_iI - D^3P_iI_c) + (T^*)^2(2D^2IE_i - 2D^2Q_{kd}{}^*P_iI - 2D^2IE_i + 2D^2Q_{kd}{}^*P_iI) + (T^*)(2D^2E_i + 2D^2A + D\pi(Q_{kd}{}^*)^2 + 2D^2SQ_{kd}{}^* + D^3P_it^2I_c + D^3P_it^2I_d - 4DQ_{kd}{}^*IE_i + DP_iI(Q_{kd}{}^*)^2 + 2DQ_{kd}{}^*IE_i) = -2IE_i(Q_{kd}{}^*)^2 \quad (16)$$

In order to obtain the economic operating policy, follow these steps:

- Calculate E_i at every level of the price offered by the supplier
- Perform initial iteration to calculate the number of expired products, Q_{kd}(equation 15) at every level of the price offered by the supplier to enter the order quantity value of the optimal product Q*.
- Check the feasibility of Q* obtained in step 2 is based on the discounts offered. If feasible proceed to step 5. If it is not feasible then go to step 4.
- If Q* is below the minimum limit of Ui, then Q* is used Ui. If Q* is above the maximum limit of Ui, then Q* is used U_{i+1}.
- Use Q_{kd}* value obtained in step 2 to determine the ordering cycle T* with equation 16.
- Use the result in steps 2 and 5 to determine the annual total variable cost (TC).
- Repeat iteration in step 2 to step 6 so that the value of the lowest TC.
- Results in step 7 will be obtained an optimum order quantity of products Q*, the quantity of expired products Q_{kd}*, and the ordering cycle T*.

NUMERICAL EXAMPLE

In this section, numerical examples was given to illustrate the proposed model and its solution procedure. Numerical examples based on the existing problems in the Supermarket Pamella. The parameters are as follows:

Suppose D = 3669, A = 2500, π = 50, t = 0.04, I = 0.1, I_c = 0.15, I_d = 0.02. The supplier sells units for 13667 in order quantity below 60, for 10115 in order quantity between 60 to 600, and for 9884 in order quantity above 600 units. Limit of expired product 6 month with delays of payment for 0.5 month. We solve the problem using heuristic approach by trial and error.
Numerical values are shown in Table-1.

Table-1. Total cost under different Q, Q_{kd} and T.

No.	Q*, unit	Q_{kd}, unit	T*, year	TC (IDR)
1.	2550	947	0,44	38083931
2.	2560	956	0,44	38083925
3.	2570	966	0,44	38083921
4.	2573	968	0,44	38083921
5.	2575	970	0,44	38083920
6.	2590	984	0,45	38083920
7.	2591	985	0,45	38083920
8.	2592	986	0,45	38083921

CONCLUSIONS

Inventori model is developed base on the existing problems in the supermarket "Pamela" considering factors such expiry, incremental discount and payment delays. Validation of the model using case T ≥ t and product data coca cola 1.5 liter in January to December of 2014 and other inventory costs. Solution model used a heuristic approach by trial and error. This model produces inventory policy in terms of how the quantity of expired goods, and when the booking is made.

Suggestions for further research can be guided as follows: incorporating other variables such as probabilistic demand, shortages by lost sales, limited warehouse capacity, expired products can be returned, expired product can be resold, anddiscount.

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